

## #13 Solving Systems of Conics

Solve each system algebraically, then graph to confirm your results.

Name Key

1.  $x = 2 + y$

$x^2 + y^2 = 100$

$(2+y)^2 + y^2 = 100$

$4+4y+y^2+y^2=100$

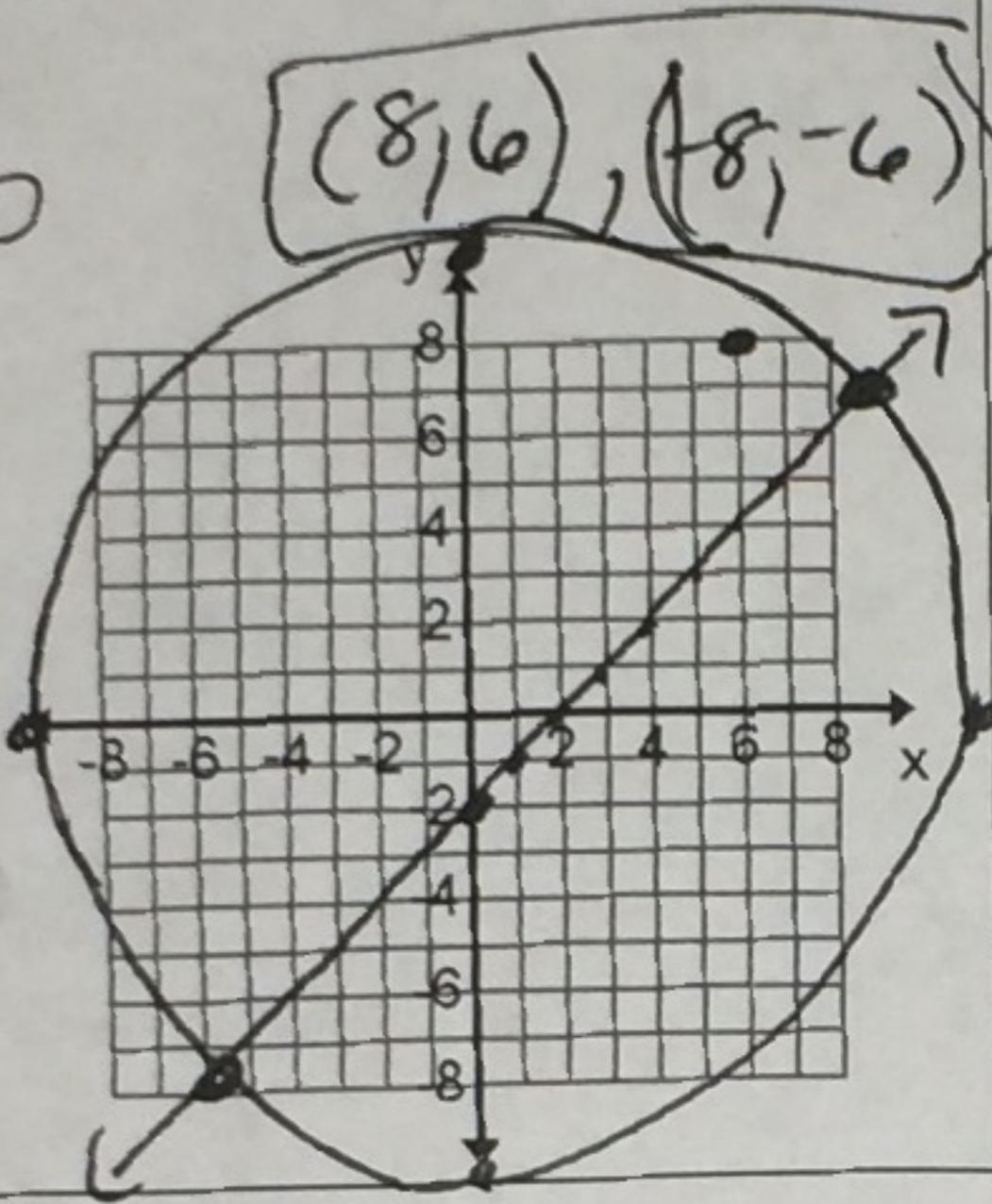
$2y^2+4y-96=0$

$y^2+2y-48=0$

$(y-6)(y+8)=0$

$y=6 \text{ or } -8$

$x = 2+6 \quad x = 2-8$   
 $x = 8 \quad x = -6$



2.  $x^2 + y^2 = 25$

$x^2 + 25y^2 = 25$

$x^2 = 25 - 25y^2$

$25 - 25y^2 + y^2 = 25$

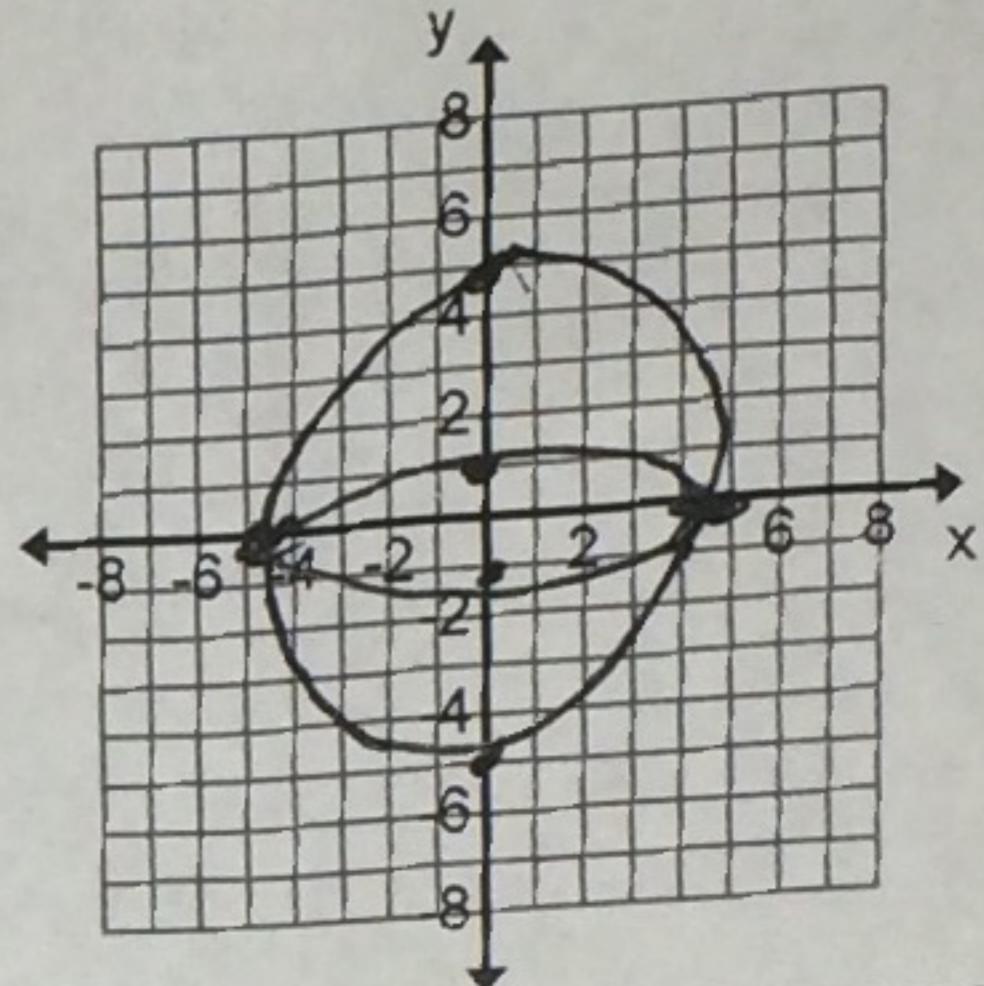
$-24y^2 = 0$

$y = 0$

$x^2 = 25$

$x = \pm 5$

$(-5, 0) (5, 0)$



3.  $4x^2 + y^2 = 4$

$y = 2x^2 - 2$

$\frac{y+2}{2} = x^2$

$4\left(\frac{y+2}{2}\right) + y^2 = 4$

$2y+4+4y^2-y=0$

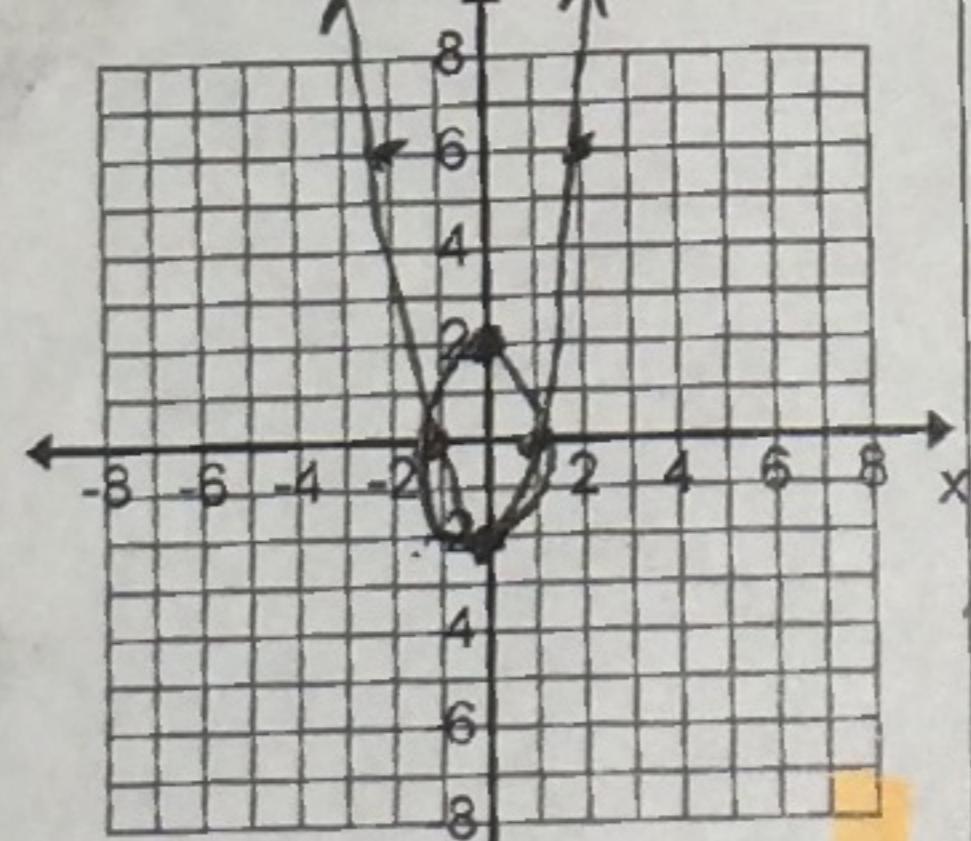
$y^2+2y=0$

$(y+2)=0$

$y=0, -2$

$0 = 2x^2 - 2$   
 $1 = x^2$   
 $1 \pm = x$

$(-1, 0) (1, 0) (-2, 0)$



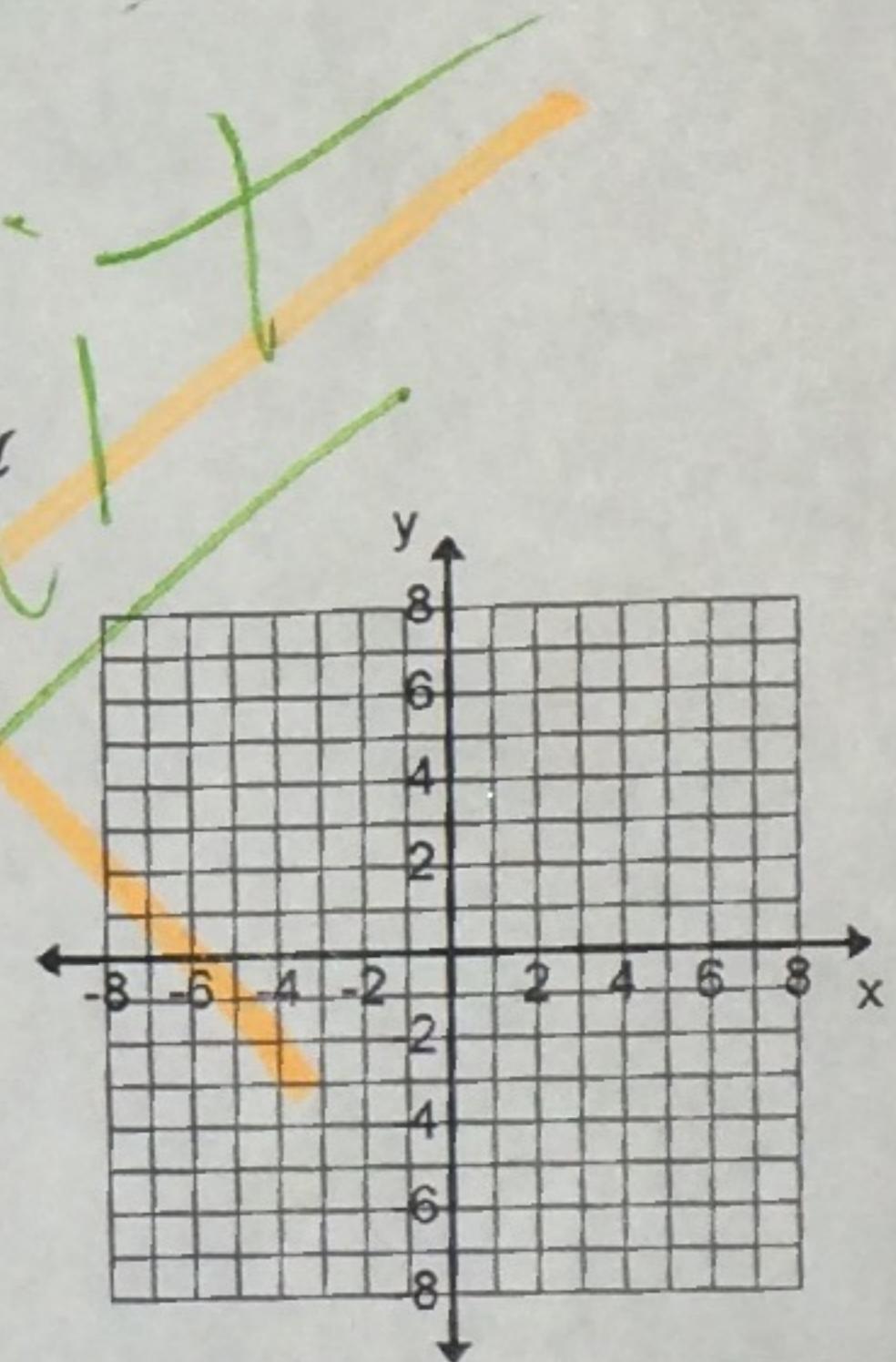
4.  $x^2 - 5y^2 = -44$

$xy = -24$

$x = \frac{-24}{y}$

$\left(\frac{-24}{y}\right)^2 - 5y^2 = -44$

$\frac{576}{y^2} - 5y^2 = -44$



5.  $16x^2 - 3y^2 = -11$

$8x - y = -11$

$y = 8x + 11$

$-8 - y = -11 \quad -16 - y = -11$

$-y = -3 \quad y = 3$

$-y = 5 \quad y = -5$

$16x^2 - 3(8x+11)^2 = -11$

$16x^2 - 3(64x^2 + 176x + 121) + 11 = 0$

$16x^2 - 192x^2 - 528x - 363 + 11 = 0$

~~$x^2 - 528$~~

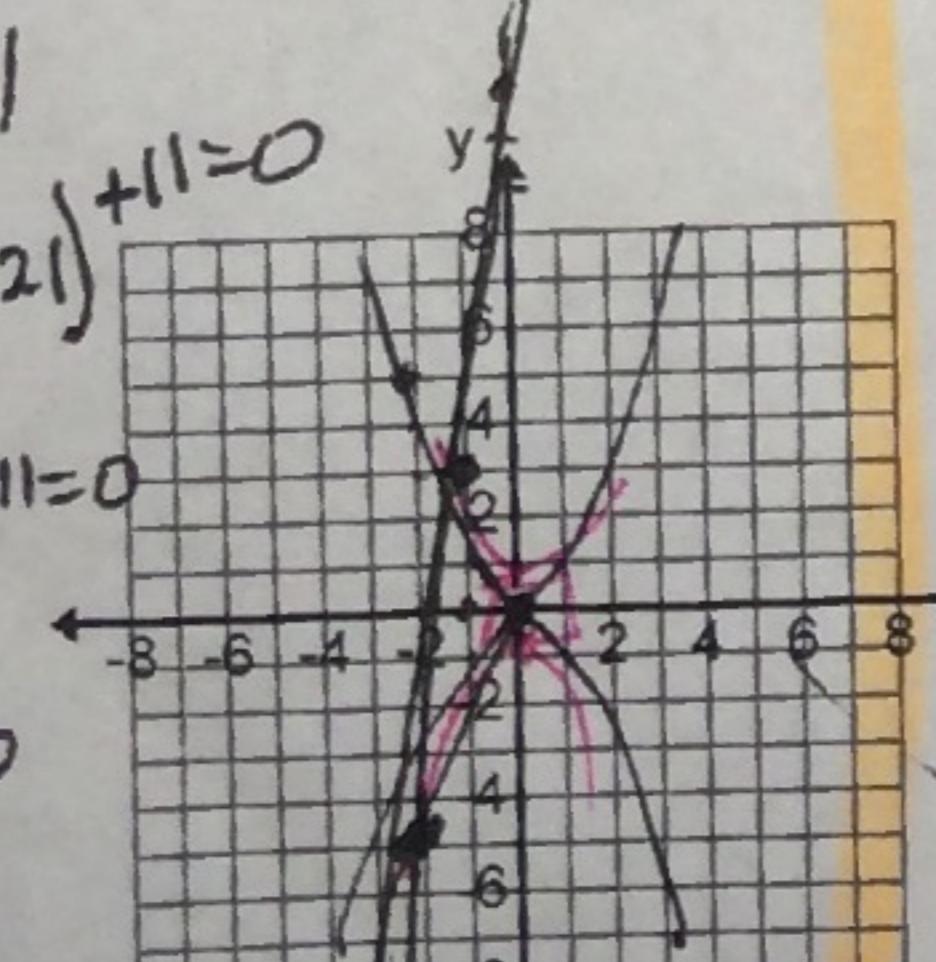
$-176x^2 - 528x - 352 = 0$

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

$x = -1 \quad x = -2$

$\frac{y^2}{3} - \frac{x^2}{16} = 1$



$(-1, 3) (-2, -5)$

6.  $2x^2 + 5y^2 = 98$

$2x^2 - y^2 = 2$

$2x^2 = y^2 + 2$

$y^2 + 2 + 5y^2 = 98$

$6y^2 = 96$

$y^2 = 16$

$y = \pm 4$

$\frac{x^2}{1} - \frac{y^2}{2} = 1$

$\frac{x^2}{49} + \frac{y^2}{45} = 1$

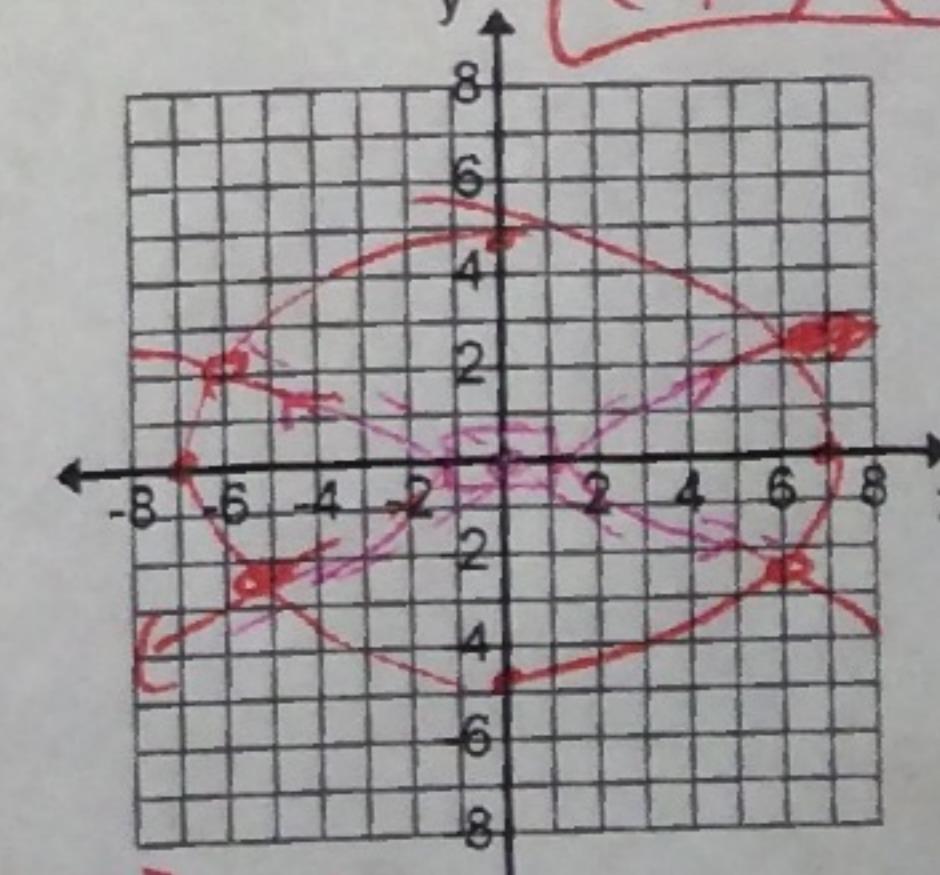
$2x^2 - 16 = 2 \quad x^2 + y^2 = 1$

$2x^2 = 18$

$x^2 = 9$

$x = \pm 3$

$(-3, 4) (-3, -4)$   
 $(3, 4) (3, -4)$



$$7. \quad x^2 + y^2 = 25$$

$$y - x^2 = -5$$

$$y+5 = x^2$$

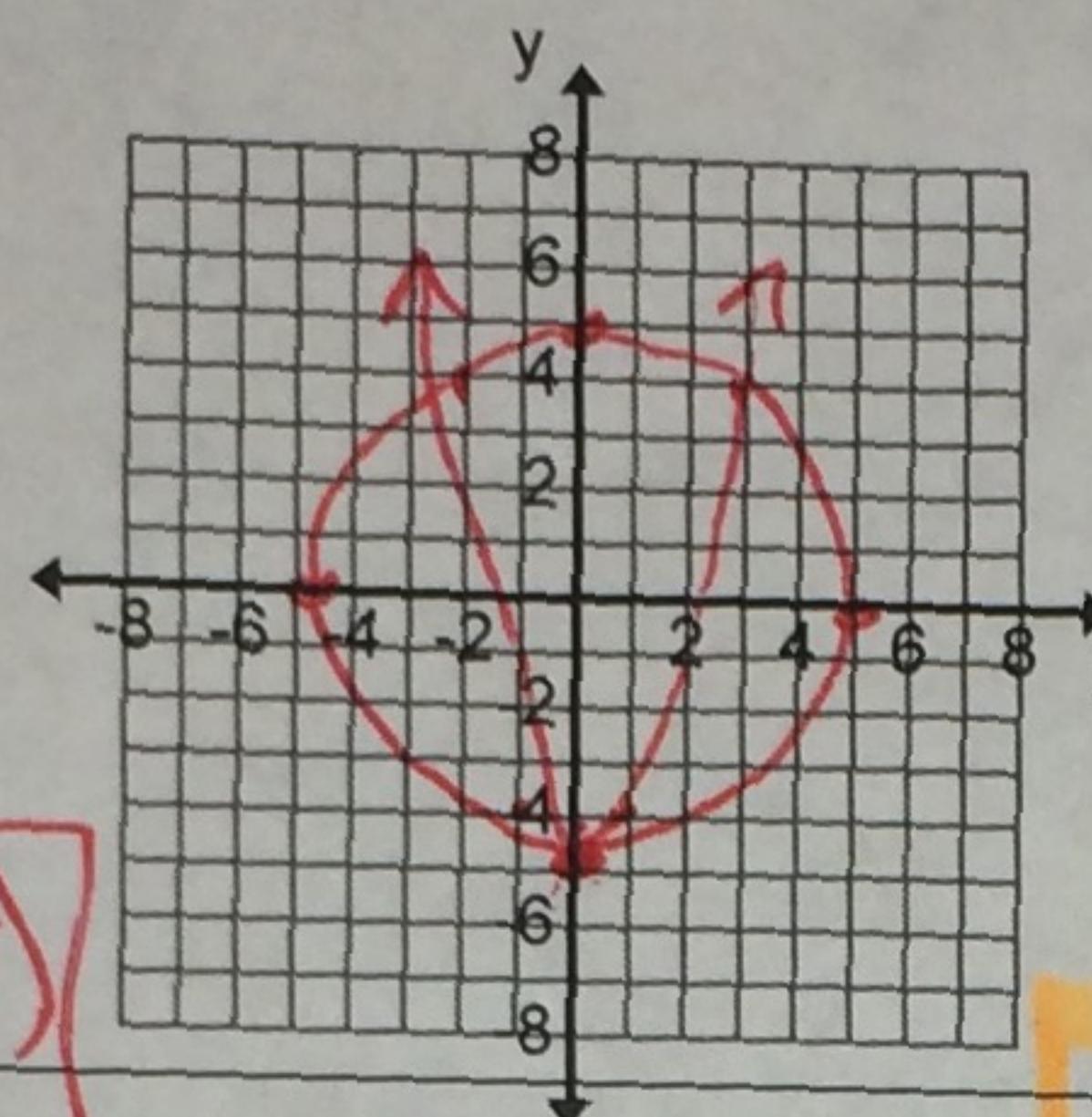
$$(y+5) + y^2 = 25$$

$$y^2 + y - 20 = 0$$

$$(y+5)(y-4) = 0$$

$$y = -5 \text{ or } 4$$

$$\boxed{(0, -5)(3, 4)(-3, 4)}$$



$$9. \quad \frac{x^2}{4} + \frac{y^2}{16} = 1$$

$$x^2 + y^2 = 1$$

$$-\frac{y^2+1}{4} + \frac{y^2}{16} = 1$$

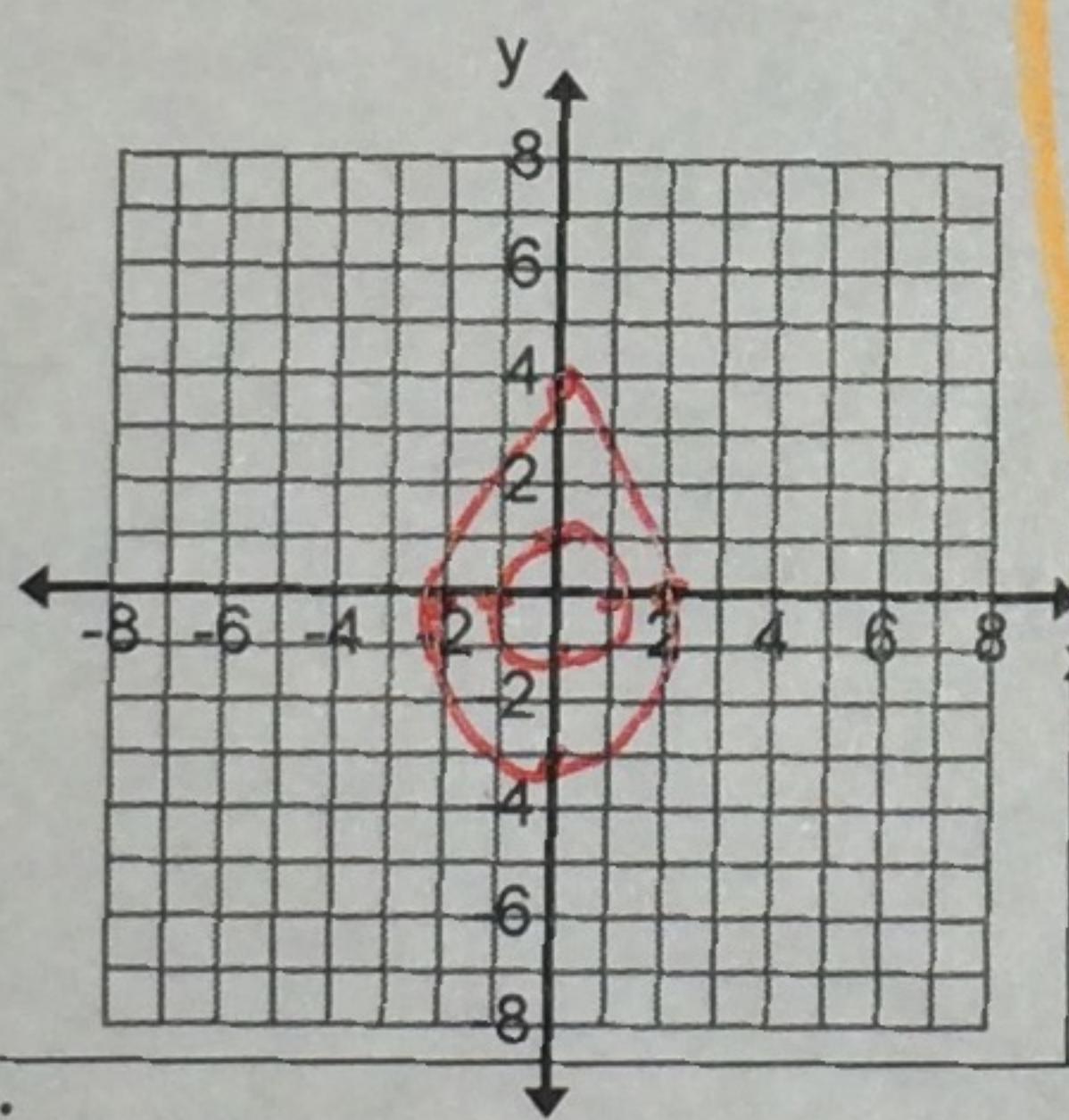
$$-\frac{4y^2+4+y^2}{16} = 1$$

$$-\frac{3y^2+4}{16} = 1$$

$$-3y^2 - 3y^2 = 16$$

$$y^2 = -4$$

N.S.



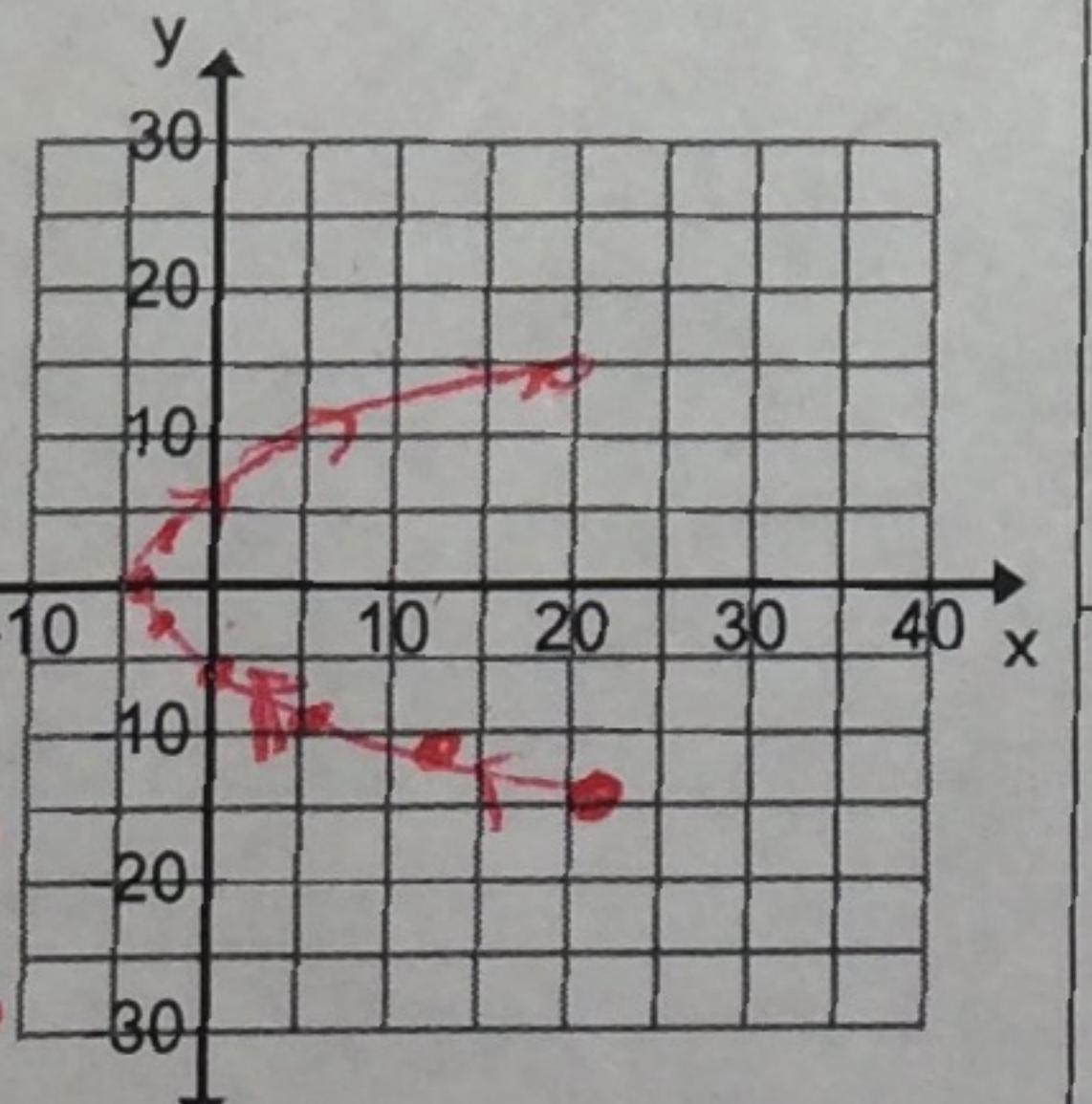
Graph the parametric equation:

$$x = t^2 - 4$$

$$11. \quad y = 3t$$

$$-5 \leq t \leq 5$$

$t$	-5	-4	-3	-2	-1	0	1
$x$	21	12	5	0	-3	-4	-3
$y$	-15	-12	-9	-6	-3	-2	-3



$$8. \quad 9x^2 - 4y^2 = 36 \quad \frac{x^2}{4} - \frac{y^2}{9} = 1$$

$$2y = x - 2$$

$$2y + 2 = x \quad y = \frac{1}{2}x - 1 \quad +2(-2, 2.25) + 2 = x$$

$$-2, 2.25 \quad \boxed{(-2, 2.25), (2, 0)}$$

$$9(2y+2)^2 - 4y^2 = 36$$

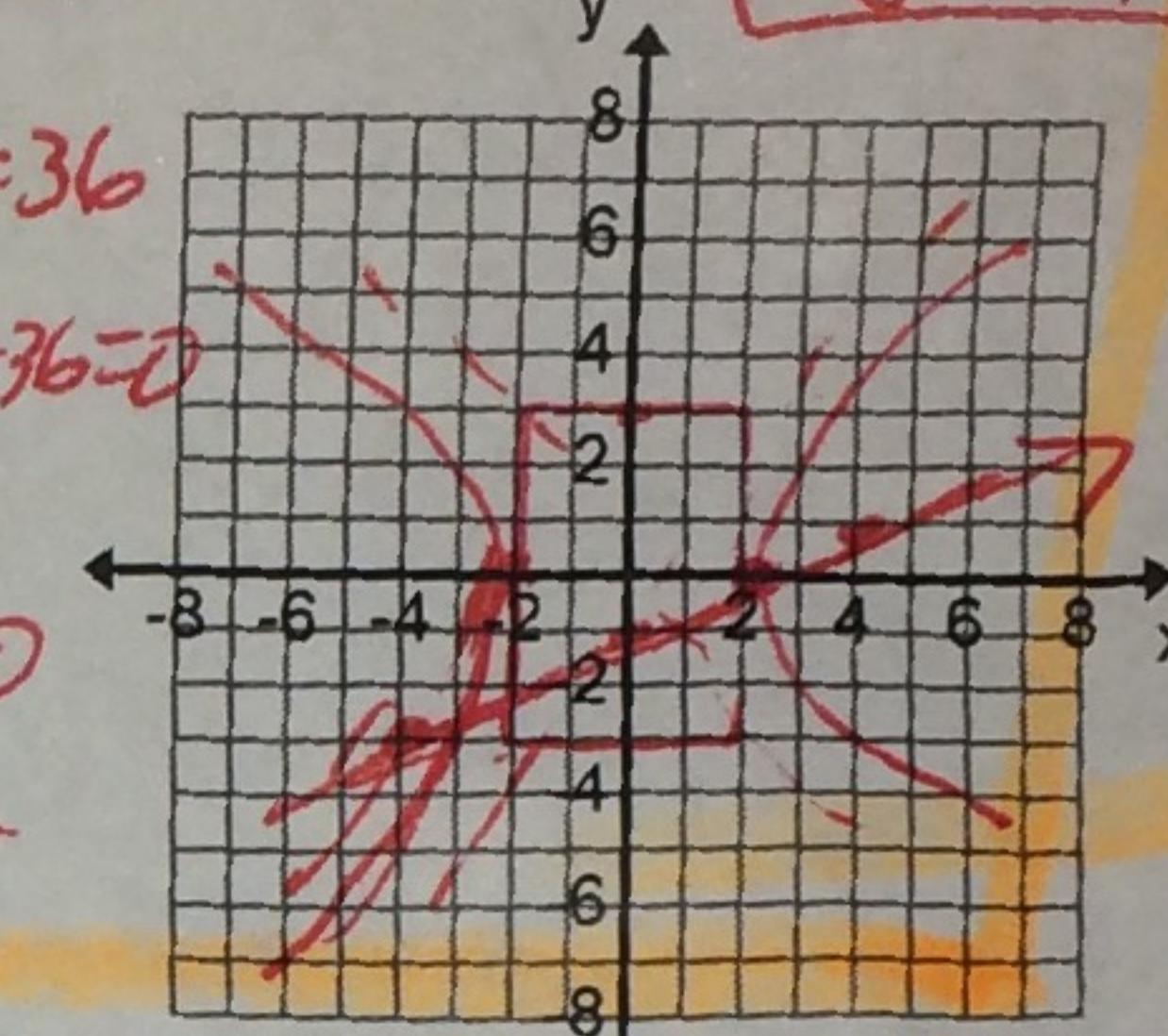
$$9(4y^2 + 8y + 4) - 4y^2 = 36$$

$$36y^2 + 72y + 36 - 4y^2 = 36$$

$$32y^2 + 72y = 0$$

$$32y(y + 2.25) = 0$$

$$y = 0 \quad y = -2.25$$



$$10. \quad x^2 + 9y^2 - 10x + 36y = 20$$

$$x - 3y = 2$$

$$X = 3y + 2$$

$$(x-5)^2 + 9(y+2)^2 = 81$$

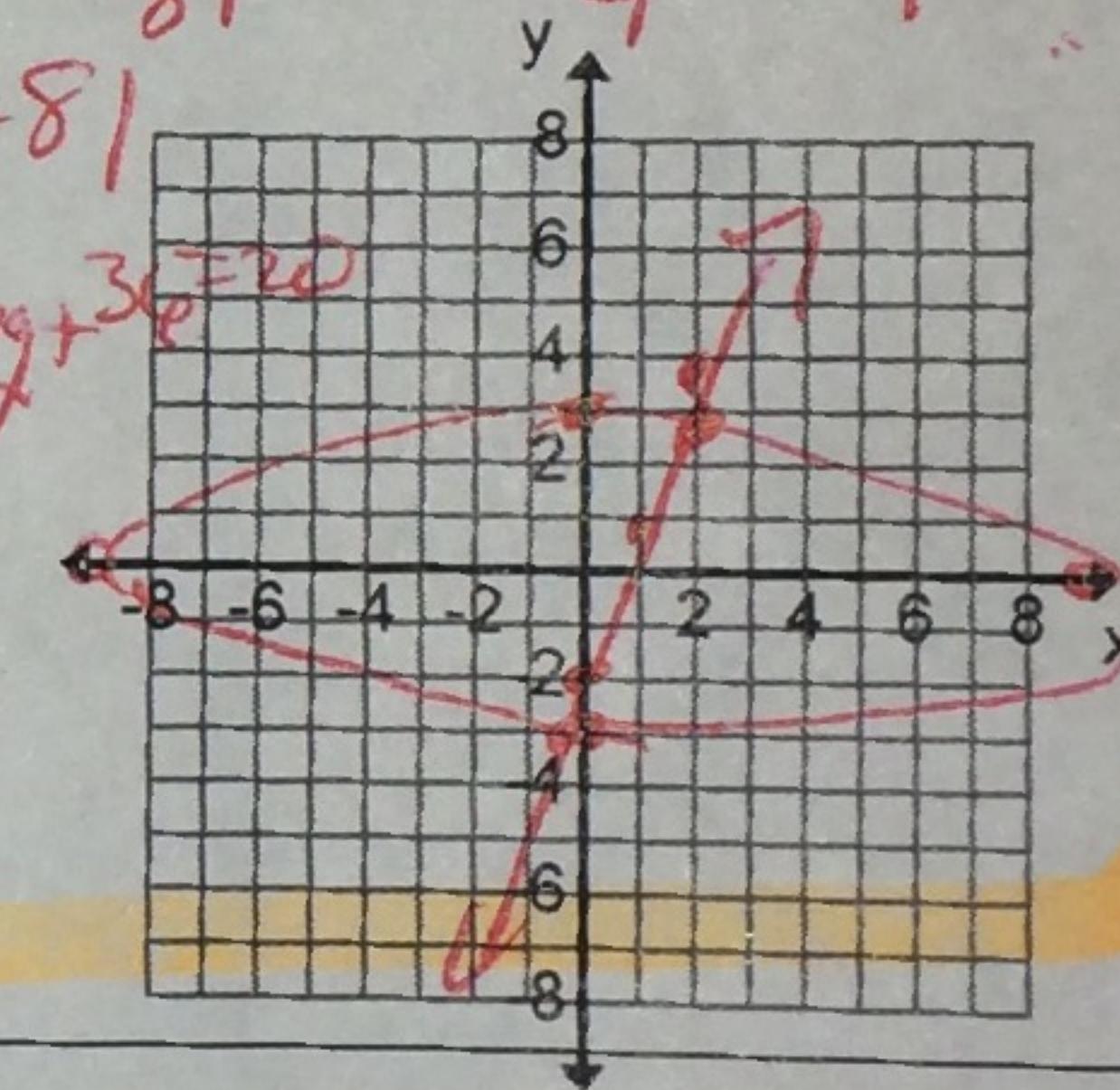
$$\frac{(x-5)^2}{81} + \frac{(y+2)^2}{9} = 1$$

$$\underline{(3y+2)^2} + 9(y+2)^2 = 81$$

$$(9y^2 + 12y + 4) + 9y^2 + 36y + 36 = 81$$

$$18y^2 + 48y + 20 = 0$$

$$2(9y^2 + 24y + 10) = 0$$



Eliminate the parameter:

$$12. \quad x = t - 4$$

$$y = t^2 + 1$$

$$t = x + 4$$

$$u = (x+4)^2 + 1$$

$$y = x^2 + 4x + 17$$

$$x = 3 \cos T + 5$$

$$y = 2 \sin T - 1$$

$$\frac{x-5}{3} = \cos T$$

$$\frac{(x-5)^2}{9} = \cos^2 T$$

$$\frac{(x-5)^2}{9} + \frac{(y+1)^2}{4} = 1$$

$$13. \quad x = 5t - 4 \quad \frac{x+4}{5} = t$$

$$y = \sqrt{t} - 1$$

$$y = \sqrt{\frac{x+4}{5}} - 1$$

$$14. \quad x = \sqrt{3} \cos T + 2$$

$$y = \sqrt{3} \sin T - 7$$

$$(x-2)^2 + (y+7)^2 = 3$$

16. Mrs. E's dog, Coco, sees a squirrel in the back yard. The squirrel and Coco are both 100 feet away from the same tree that the squirrel is attempting to take refuge in, not knowing Coco just wants to play and race to the tree. Coco:  $x_1 = 16(t-1)$ ;  $y_1 = 2$  Squire:  $x_1 = 12t$ ;  $y_1 = 3$

After 3 seconds, \_\_\_\_\_ is ahead by \_\_\_\_\_ feet.

After 7 seconds, \_\_\_\_\_ is ahead by \_\_\_\_\_ ft.

\_\_\_\_\_ wins the race by \_\_\_\_\_ feet with a winning time of \_\_\_\_\_ seconds.