

#12 WS Parametric Equations

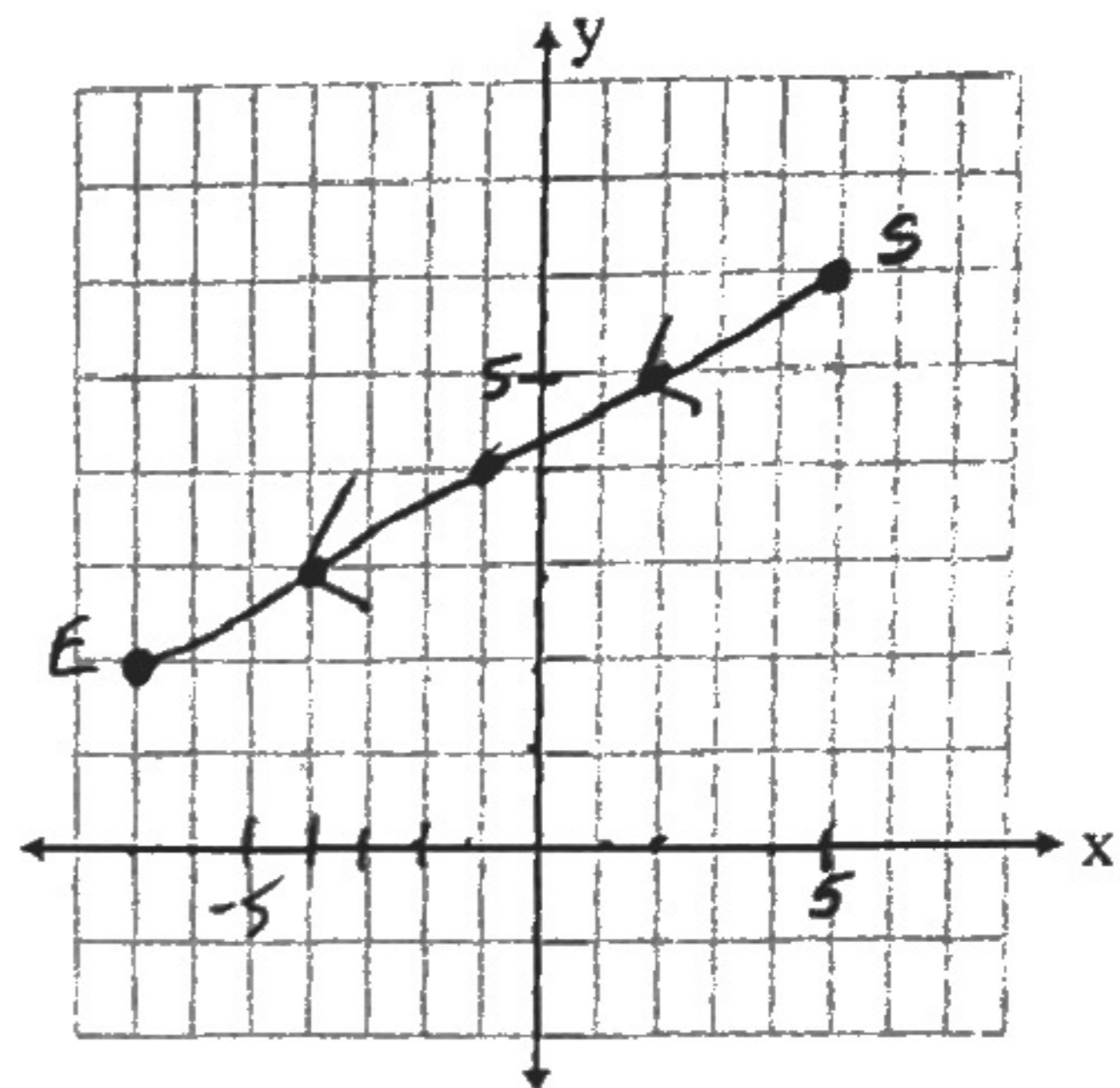
Name Key

Fill in the requested information AND sketch the curve for EACH pair of parametric equations.

1. $x(t) = 2 - 3t$ and $y(t) = 5 - t$ for t in $[-1, 3]$

t	-1	0	1	2	3
x	5	2	-1	-4	-7
y	6	5	4	3	2

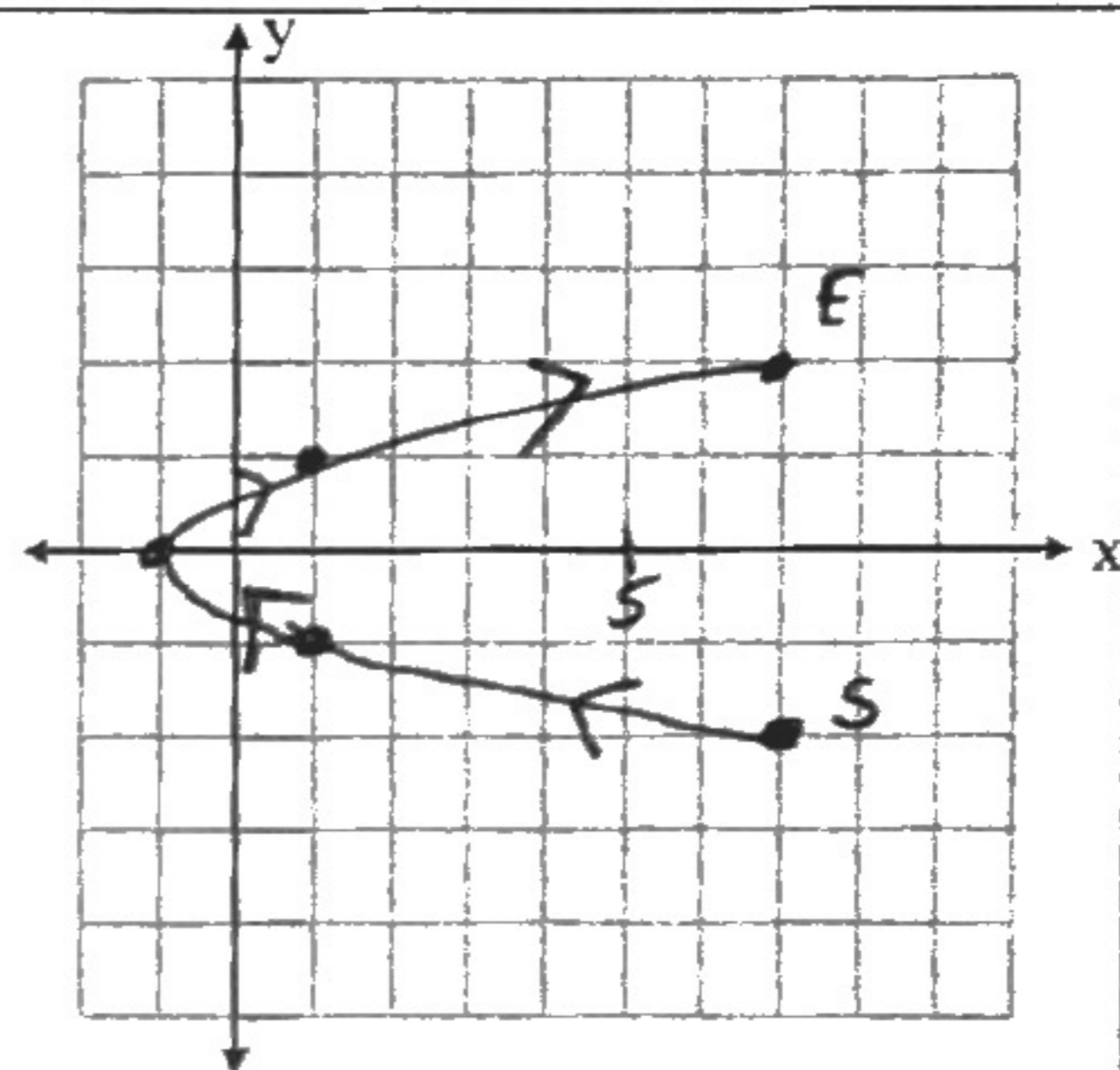
Endpoints of the Curve: (5, 6) and (-7, 2)
 Domain: [-7, 5] Range: [2, 6]
 Does the curve represent a function? yes
 Why or why not? Passes VLT



2. $x(t) = 2t^2 - 1$ and $y(t) = t$ for t in $[-2, 2]$

t					
x					
y					

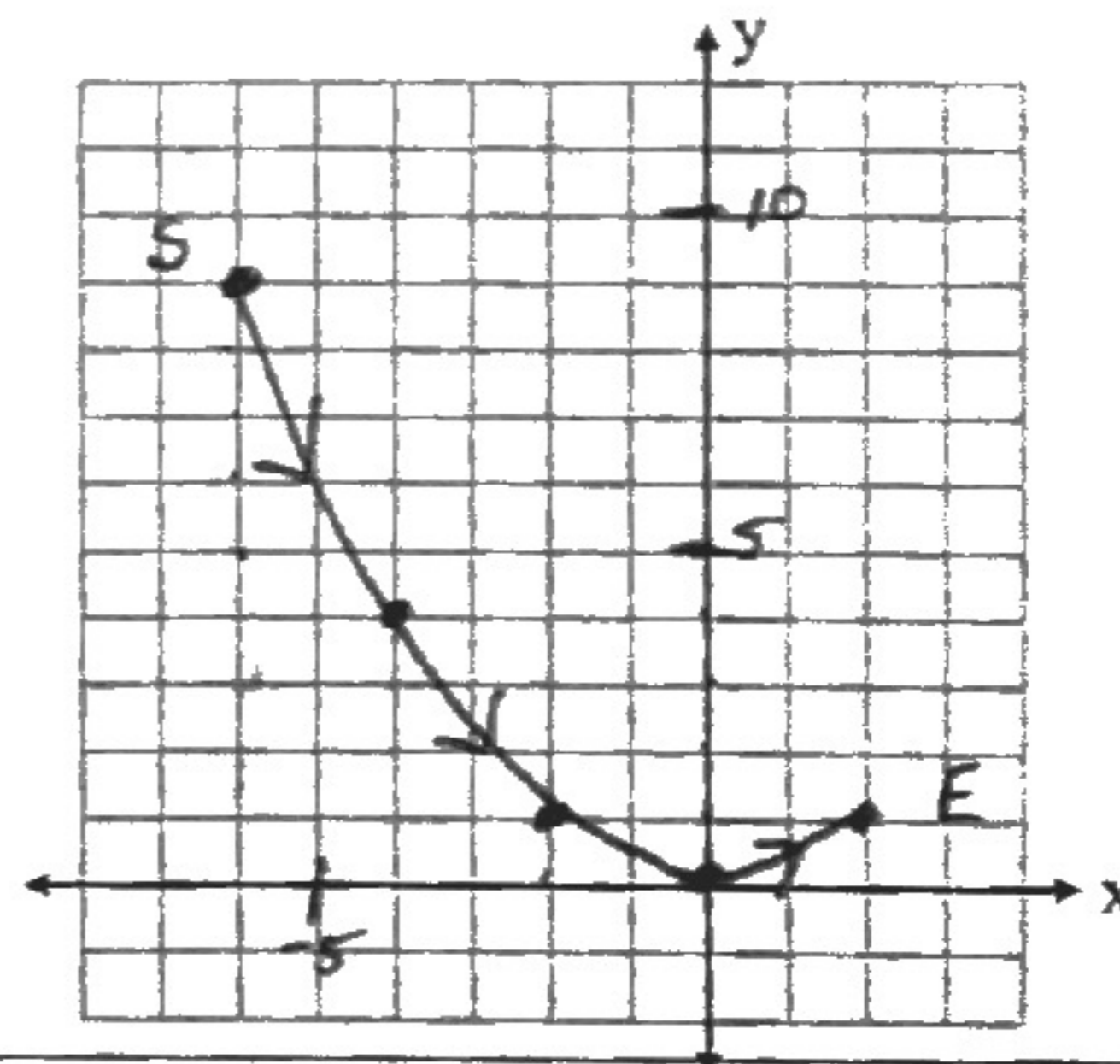
Endpoints of the Curve: _____ and _____
 Domain: _____ Range: _____
 Does the curve represent a function? _____
 Why or why not? _____



3. $x(t) = 2t$ and $y(t) = t^2$ for t in $[-3, 1]$

t	-3	-2	-1	0	1
x	-6	-4	-2	0	2
y	9	4	1	0	1

Endpoints of the Curve: (-6, 9) and (2, 1)
 Domain: [-6, 2] Range: [0, 9]
 Does the curve represent a function? yes
 Why or why not? Passes VLT



Convert from Parametric to Rectangular by eliminating the parameter.

4. $x = 2t - 5$
 $y = t^2 + 4$

$y = \frac{1}{4}(x+5)^2 + 4$

5. $x = t^2 - 2$
 $y = 5t$
 $\frac{y}{5} = t$
 $x = \left(\frac{y}{5}\right)^2 - 2$
 $x = \frac{y^2}{25} - 2$

6. $x = -t - 4$
 $y = 3t^2$

$y = 3(-x-4)^2$

<p>7. $x = \frac{t}{3} + 2$ $y = \frac{t^2}{6} - 7$</p> <p>$3(x-2) = t$</p> <p>$y = \frac{[3(x-2)]^2}{6} - 7$</p> <p>$y = \frac{9(x-2)^2}{6} - 7$</p> <p>$y = \frac{3}{2}(x-2)^2 - 7$</p>	<p>8. $x = 2 \cos T$ $y = 5 \sin T$</p> <p>8.5</p> <p>$\frac{x^2}{4} + \frac{y^2}{25} = 1$</p>	<p>9. $x = 6 \cos T + 5$ $y = 6 \sin T - 1$</p> <p>$\cos T = \frac{x-5}{6}$ $\sin T = \frac{y+1}{6}$</p> <p>$\cos^2 T + \sin^2 T = 1$ $\frac{(x-5)^2}{36} + \frac{(y+1)^2}{36} = 1$</p> <p>$(x-5)^2 + (y+1)^2 = 36$</p>
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<p>10. $x = T$ $y = -2T^2 + 3$</p> <p>$y = -2x^2 + 3$</p>	<p>11. $x = 3 \cos T + 2$ $y = \sin T + 5$</p> <p>$\frac{(x-2)^2}{9} + \frac{(y-5)^2}{1} = 1$</p>	<p>12. $x = \frac{2}{\cos T} + 1$ $x = 2 \sec T + 1$ $y = 5 \sin T - 4$</p> <p><u>oops</u></p>
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Write an equation in Standard form for each:

<p>13. A circle with a center at (-3,8) and tangent to the y-axis.</p> <p>$(x+3)^2 + (y-8)^2 = 9$</p>	<p>14. A parabola with a focus at (-1,2) and a directrix at x=5. Show work!</p> <p>$x = -\frac{1}{12}(y-2)^2 + 2$</p>	<p>15. An ellipse with vertices at (-2,2) and (-2,8) and the length of the minor axis is 4.</p> <p>$\frac{(x+2)^2}{4} + \frac{(y-5)^2}{9} = 1$</p>
<p>16. A hyperbola with a transverse axis endpoint of (3,2) and a conjugate axis endpoint of (-4,4).</p> <p>$\frac{(x+4)^2}{49} - \frac{(y-2)^2}{4} = 1$</p>	<p>17. A hyperbola with asymptotes of $y-3 = \pm \frac{4}{3}(x-5)$</p> <p>$\frac{(x-5)^2}{16} - \frac{(y-3)^2}{9} = 1$ OR $\frac{(y-3)^2}{16} - \frac{(x-5)^2}{9} = 1$</p>	<p>18. An ellipse with vertices of (-2,-4) and (10,-4) and foci of $(2-\sqrt{39}, -4)$ and $(2+\sqrt{39}, -4)$.</p> <p>$\frac{(x-2)^2}{64} + \frac{(y+4)^2}{25} = 1$</p>

Application Problems.

19. Ben can sprint at the rate of 24 feet per second. Jerry sprints at 20 feet per second. Ben gives Jerry a 10-foot head start. Write the parametric equations that can be used to model the race. Find a viewing window to simulate a 100-yard dash. Who is ahead after 2 seconds and by how many feet? Who is ahead after 2.5 seconds and by how many feet? Who is ahead after 3 seconds and by how many feet? Who wins the race and by how many feet? What is the winning time?

BEN: $x_1 = 24t$ and $y_1 = 1$ *any thing*

JERRY: $x_2 = 20t + 10$ and $y_2 = 3$ *300ft*

Window:
 $t_{\min} = 0$ $t_{\max} = 15$ $t_{\text{step}} = .01$
 $x_{\min} = 0$ $x_{\max} = 300$ $x_{\text{step}} = 10$
 $y_{\min} = 0$ $y_{\max} = 5$ $y_{\text{step}} = 1$

After 2 seconds, Jerry is ahead by 2 feet. After 2.5 seconds, tied is ahead by Same feet. After 3 seconds, Ben is ahead by 2 feet. Ben wins the race by 40 feet with a winning time of 12.5 seconds.