Pre-Calculus Worksheet

#2 Geometric Sequence and Partial Sums

Name: Key

Period: ____

Determine whether the sequence is geometric. If so, find the common ratio.

1. $5,15,45,135,\ldots$ Geometric $r=5$	2. $36,27,18,9,\ldots$ Arithmetic $d=-9$	3. $1,-\frac{1}{2},\frac{1}{4},-\frac{1}{8},\ldots$ Geometric $r=-\frac{1}{2}$	4. $9,-6,4,-\frac{8}{3},\ldots$ Geometric, $r=-\frac{2}{3}$

Find the **recursive formula** for a_n.

Find the **explicit formula** for a_n.

5. $a_1=-4$ and $r=\frac{4}{3}$ $a_1=4$ $a_N=\frac{4}{3}a_{N-1}$	6. $a_n=3(4)^{n-1}$ $a_1=3$ $a_N=4a_{N-1}$	7. $a_1=6$ and $r=-4$ $a_N=6(-4)^{N-1}$	8. $a_1=5$, $a_{n+1}=6a_n$ $a_N=5(6)^{N-1}$

Find the **explicit formula** for a_n given the geometric sequence. Then find a_{10}.

8. $a_1=5$ and $r=3$ $\boxed{a_N=5(3)^{N-1}}$ $a_{10}=5(3)^{10-1}$ $\boxed{a_{10}=98{,}415}$	9. $-2,-6,-18,-54,\ldots$ $r=3$ $a_1=-2$ $\boxed{a_N=-2(3)^{N-1}}$ $a_{10}=-2(3)^{10-1}$ $\boxed{a_{10}=-39{,}366}$	10. $a_1=4$ and $\boxed{a_{10}=\frac{1}{128}}$ $a_{10}=a_1 r^{10-1}$ $\frac{1}{128}=4(r)^9$ $\frac{1}{512}=r^9$ $\frac{1}{2}=r$ $\boxed{a_N=4(\frac{1}{2})^{N-1}}$
11. $a_5=\frac{81}{5}$ and $a_8=\frac{-2187}{5}$ $a_8=a_5(r)^{8-5}$ $\frac{-2187}{5}=\frac{81}{5}r^3$ $-27=r^3$ $-3=r$ $a_5=a_1(-3)^{5-1}$ $\frac{81}{5}=a_1(-3)^4$ $\frac{1}{5}=a_1$ $\boxed{a_N=\frac{1}{5}(-3)^{(N-1)}}$	12. $\frac{2}{3},\frac{8}{9},\frac{32}{27},\frac{128}{81},\ldots$ $r=\frac{4}{3}$ $a_1=\frac{2}{3}$ $\boxed{a_N=\frac{2}{3}(\frac{4}{3})^{N-1}}$ $a_{10}=\frac{2}{3}(\frac{4}{3})^9$ $a_{10}\approx 8.879$	13. $a_3=20$ and $r=2$ $a_3=a_1(2)^{3-1}$ $20=a_1(2)^2$ $5=a_1$ $\boxed{a_N=5(2)^{N-1}}$ $a_{10}=5(2)^9$ $\boxed{a_{10}=2560}$

Find the indicated number of geometric means for each:

14. Geometric mean between -18 and -36. 1 2 3 -18 ___ -36 $a_3=a_1(r)^{3-1}$ $-36=-18(r)^2$ $2=r^2$ $\pm\sqrt{2}=r$ $\boxed{-18\sqrt{2} \text{ OR } 18\sqrt{2}}$	15. Insert ~~1~~ 2 geometric means between -4 and 108 1 2 3 4 $-4,\_,\_,+108$ $+108=-4(r)^3$ $-27=r^3$ $-3=r$ $\boxed{12,-36}$	16. Insert 4 geometric means between 1 and 2 $1,\_,\_,\_,\_,2$ $a_6=a_1 r^{6-1}$ $2=1r^5$ $\sqrt[5]{2}=r$ $2^{\frac{1}{5}}=r$ $\boxed{2^{\frac{1}{5}},2^{\frac{2}{5}},2^{\frac{3}{5}},2^{\frac{4}{5}}}$

Write the first five terms of the sequence.
(Assume that n begins with 0.)

17. $a_n = \frac{3^n}{n!}$ $\quad 0! = 1$

0, 1, 2, 3, 4

$\boxed{1, 3, \frac{9}{2}, \frac{9}{2}, \frac{27}{8}}$

18. $a_n = \frac{1}{(n+1)!}$

0, 1, 2, 3, 4

$\boxed{1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}}$

Simplify the factorial expression.

19. $\frac{5!}{8!} = \frac{1}{8\cdot 7\cdot 6} = \boxed{\frac{1}{336}}$

20. $\frac{(n+1)!}{n!}$

$\frac{(N+1)(N)\cancel{6}\ldots 1}{N\ldots 1}$

$\boxed{N+1}$

Find the sum WITHOUT the calculator.

21. $\sum_{i=1}^{5}(2i+1)$

$= [2(1)+1]+[2(2)+1]+[2(3)+1]+[2(4)+1]+[2(5)+1]$

$= 3+5+7+9+11$

$= 35$

22. $\sum_{k=1}^{4}10$

$= 10+10+10+10$

$= 40$

23. $\sum_{j=0}^{4}j^2$

$= 0^2+1^2+2^2+3^2+4^2$

$= 0+1+4+9+16$

$= 30$

Find the sum WITH the calculator. Write down what you type into the calculator, please.

24. $\sum_{i=1}^{6}(24-3i) = 81$

25. $\sum_{j=1}^{10}\left(\frac{3}{j+1}\right) = \frac{55991}{9240}$

≈ 6.0596

26. $\sum_{k=0}^{4}\frac{(-1)^k}{k!} = \frac{3}{8}$

Rewrite the sum using **sigma notation**.

27. $\frac{1}{3(1)}+\frac{1}{3(2)}+\frac{1}{3(3)}+\ldots+\frac{1}{3(9)}$

$\sum_{N=1}^{9}\frac{1}{3N}$

28. $\frac{5}{1+1}+\frac{5}{1+2}+\frac{5}{1+3}+\ldots+\frac{5}{1+15}$

$\sum_{N=1}^{15}\frac{5}{1+N}$

29. $3-9+27-81+243-729$ $\quad N'$

$\sum_{N=1}^{6}$

30. $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots-\frac{1}{128}$

$\sum_{N=1}^{8}\left(-\frac{1}{2}\right)^{N-1}$

Geom

$a_N = 1\left(-\frac{1}{2}\right)^{N-1}$

$-\frac{1}{128} = \left(-\frac{1}{2}\right)^{N-1}$

$\left(-\frac{1}{2}\right)^7 = \left(-\frac{1}{2}\right)^{N-1}$

$7 = N-1$

$8 = N$

31. $\left[2\left(\frac{1}{8}\right)+3\right]+\left[2\left(\frac{2}{8}\right)+3\right]+\ldots+\left[2\left(\frac{8}{8}\right)+3\right]$

$\sum_{N=1}^{8}\left[2\left(\frac{N}{8}\right)+3\right]$

32. $\left[1-\left(\frac{1}{6}\right)^2\right]+\left[1-\left(\frac{2}{6}\right)^2\right]+\ldots+\left[1-\left(\frac{7}{6}\right)^2\right]$

$\sum_{N=1}^{7}\left[1-\left(\frac{N}{6}\right)^2\right]$