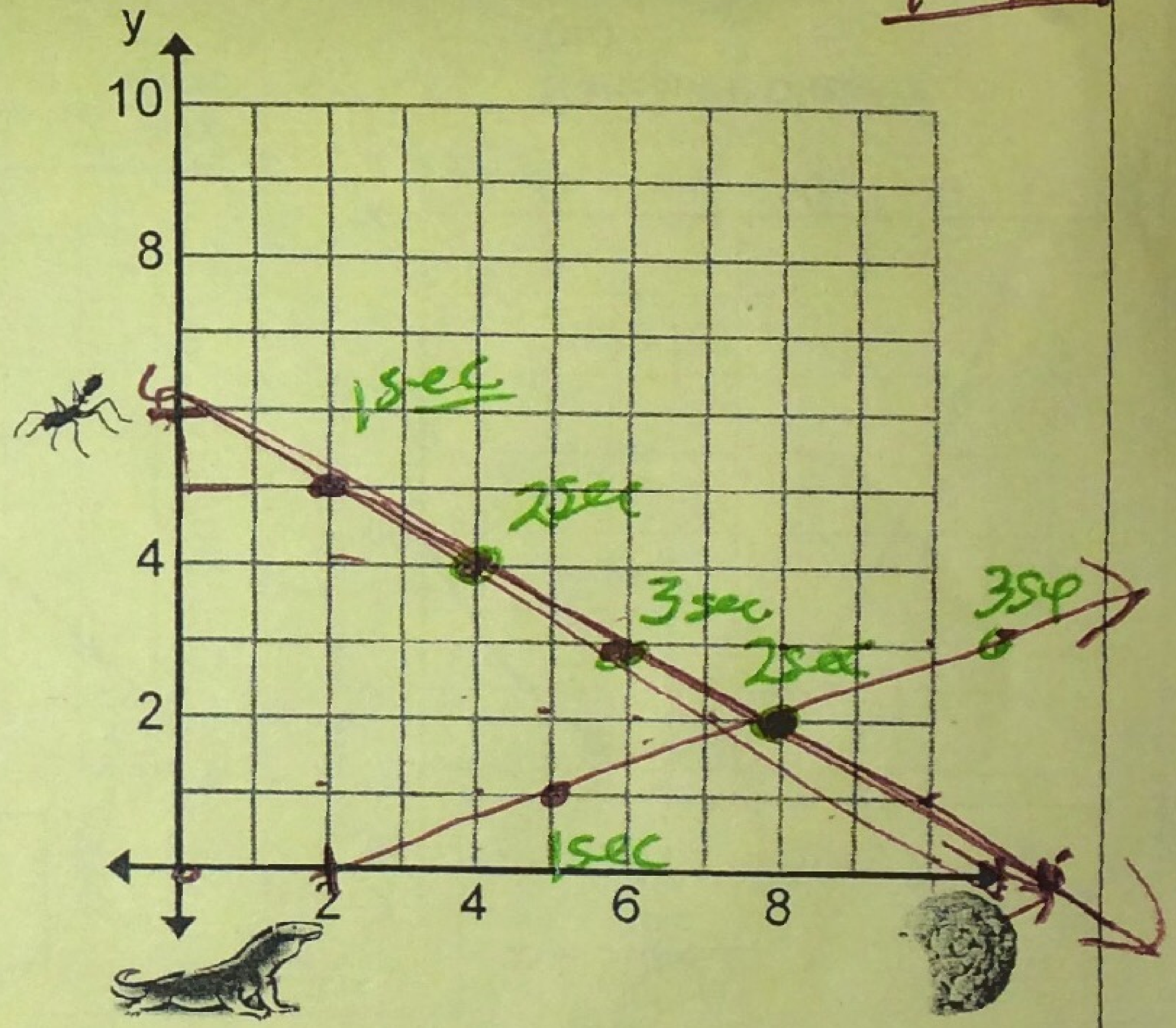
101

Parametric Equations

An ant in the corner of a wall 6 inches from the floor sees a cookie on the floor, ~~10~~ 12 inches from the corner. The ant wants the cookie and starts toward the cookie in a straight line at a rate of 1 vertical inch per 2 horizontal inches per second. A lizard, 2 inches from the corner, on the floor, traveling in a straight line can travel at a rate of 1 vertical inch per 3 horizontal inches in a second. Will the lizard "intersect" the ant at just the right time for dinner?



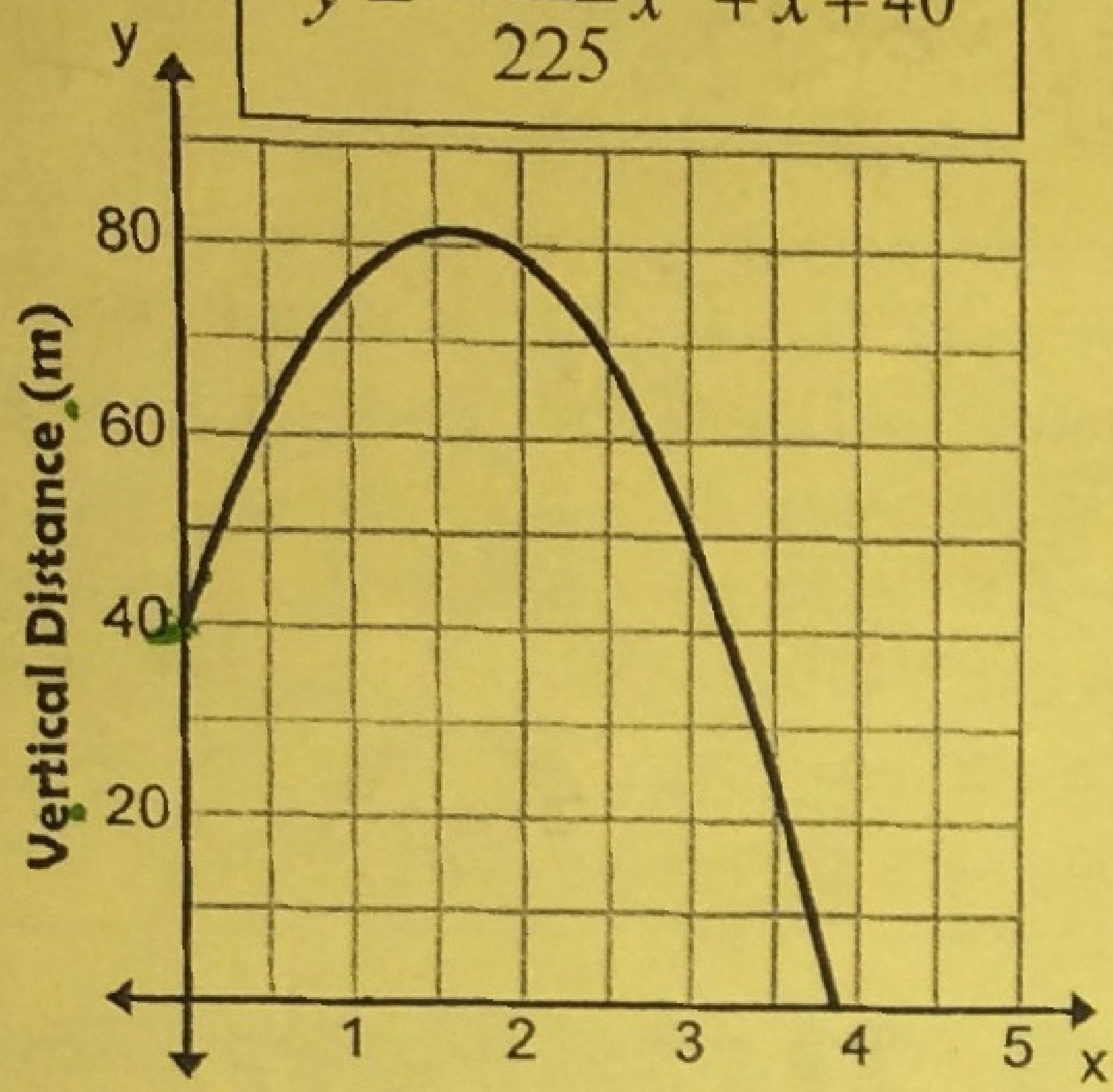
Ant

$y = -\frac{1}{2}x + 6$

Lizar

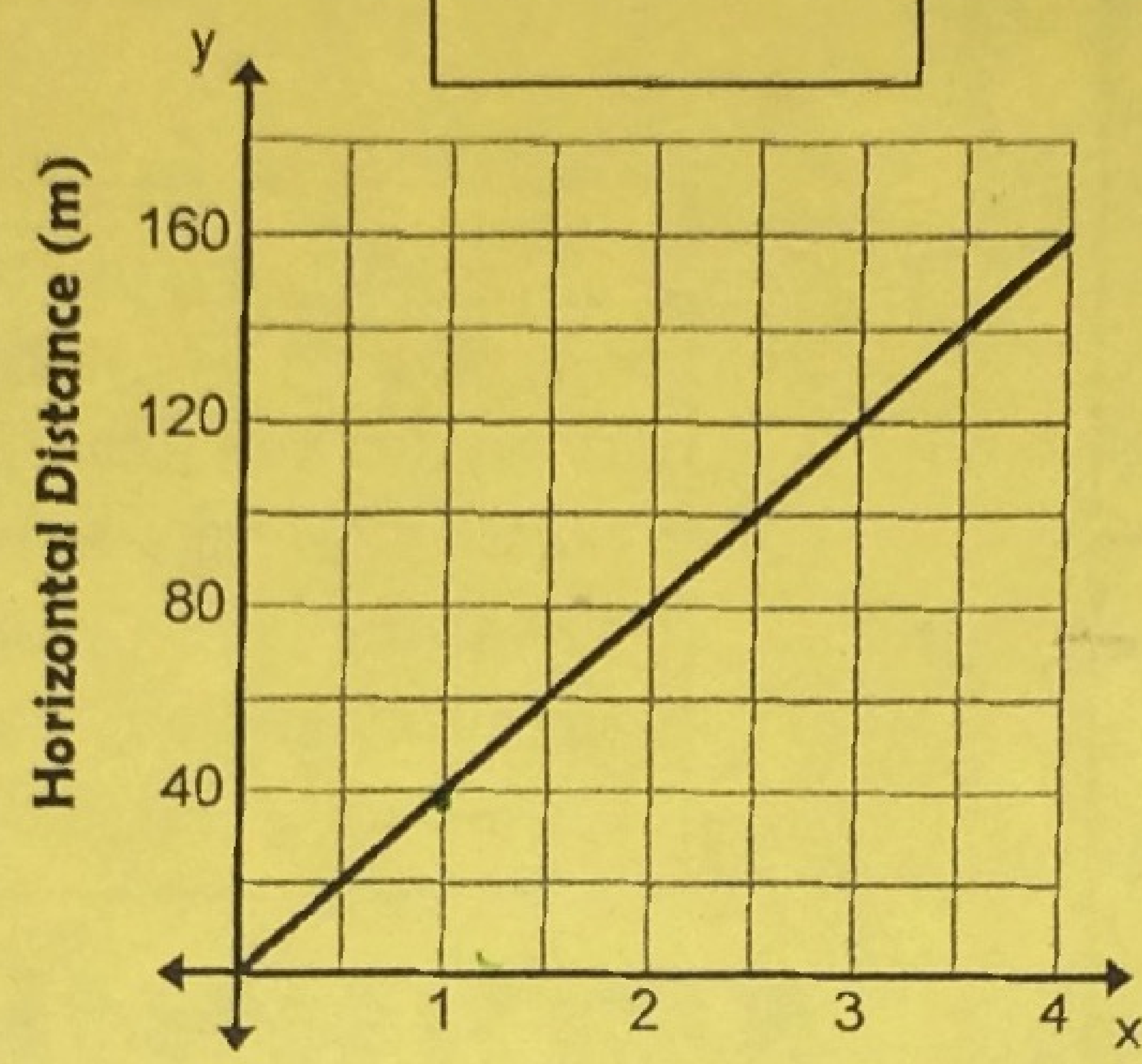
Algebra is really about relationships. **Parametric equations get us closer to the real-world relationship.** Consider the graphs below, each models a different aspect of what happens when a certain object is thrown into the air. Figure 1 shows the vertical distance the object travels as a function of time, while figure 2 shows the object's horizontal distance as a function of time. Figure 3 shows the object's vertical distance as a function if its horizontal distance.

$$y = -\frac{2}{225}x^2 + x + 40$$



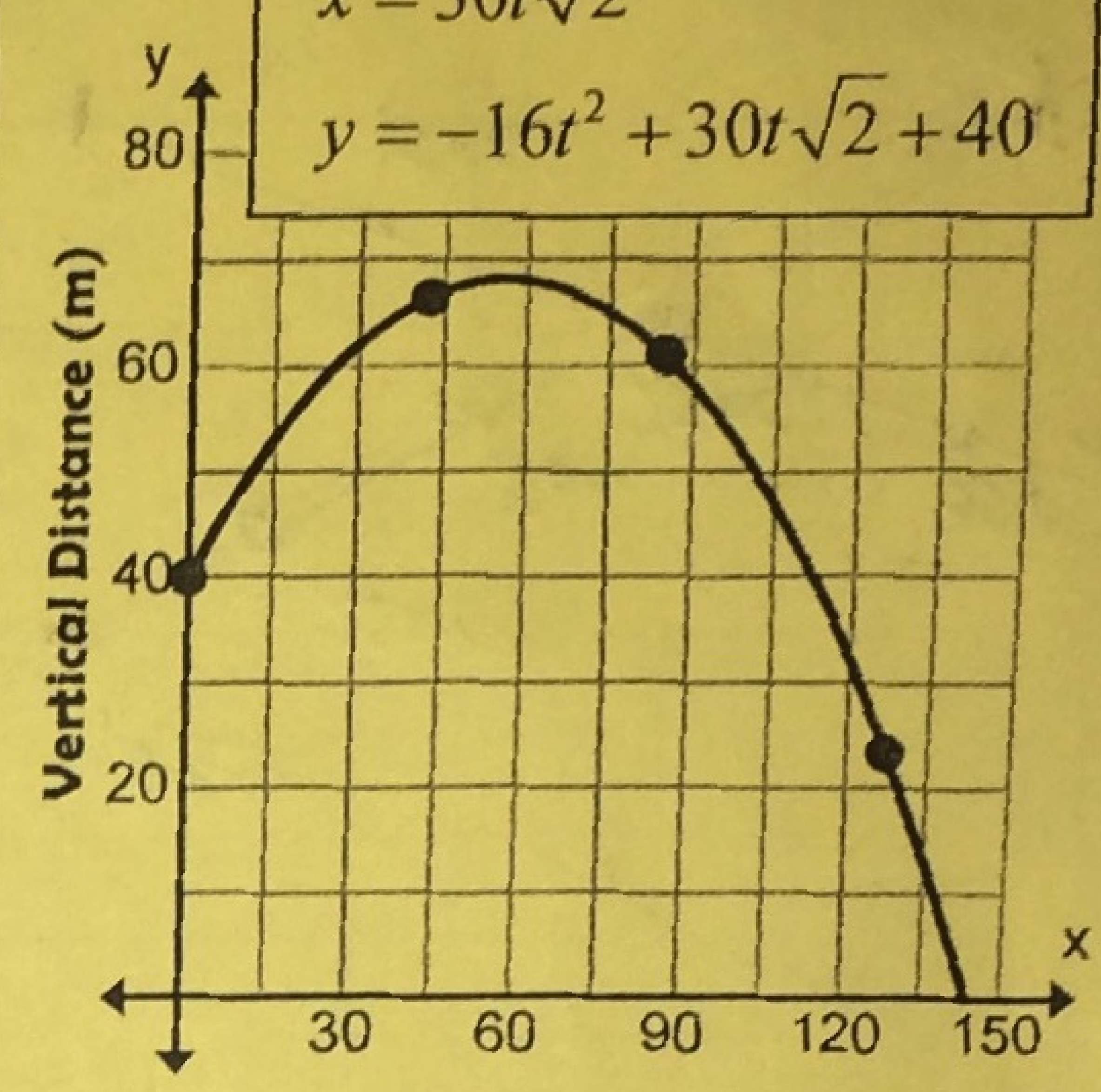
Time (s)
Figure 1

$$y = 40x$$



Time (s)
Figure 2

$$x = 30t\sqrt{2}$$
$$y = -16t^2 + 30t\sqrt{2} + 40$$

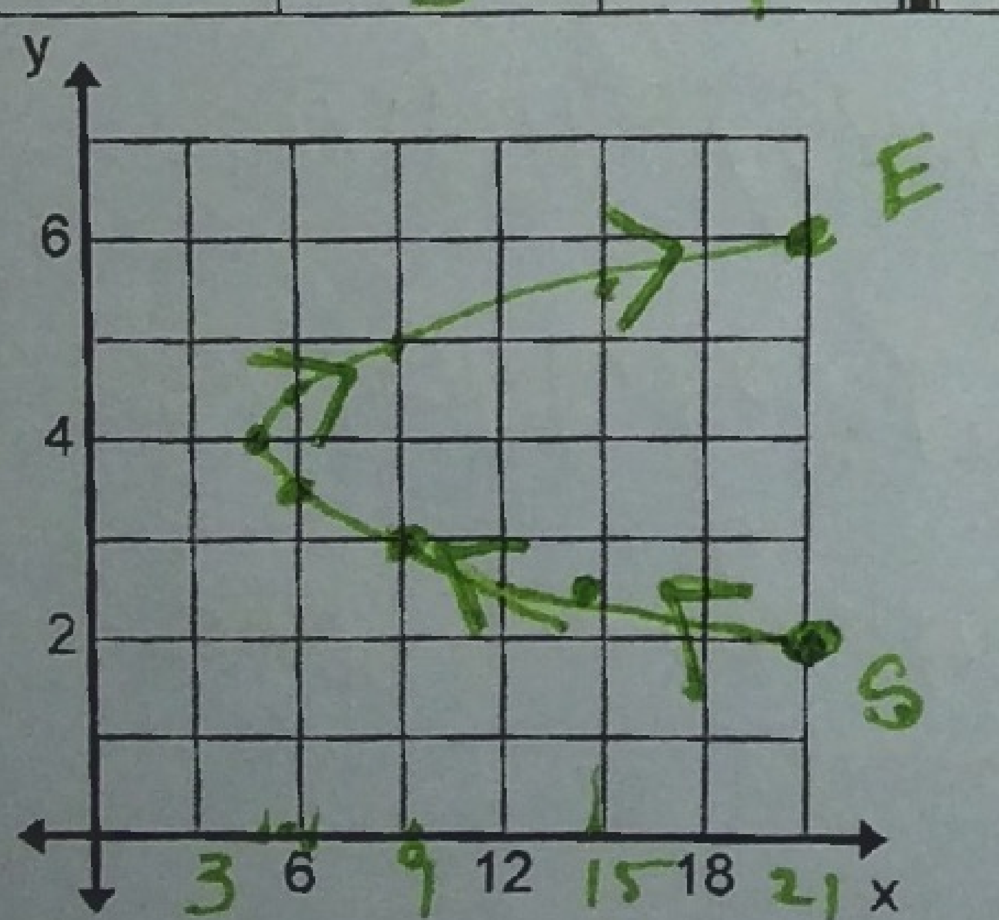


Horizontal Distance (m)

Sketch the curve given by each pair of parametric equation over the given interval.

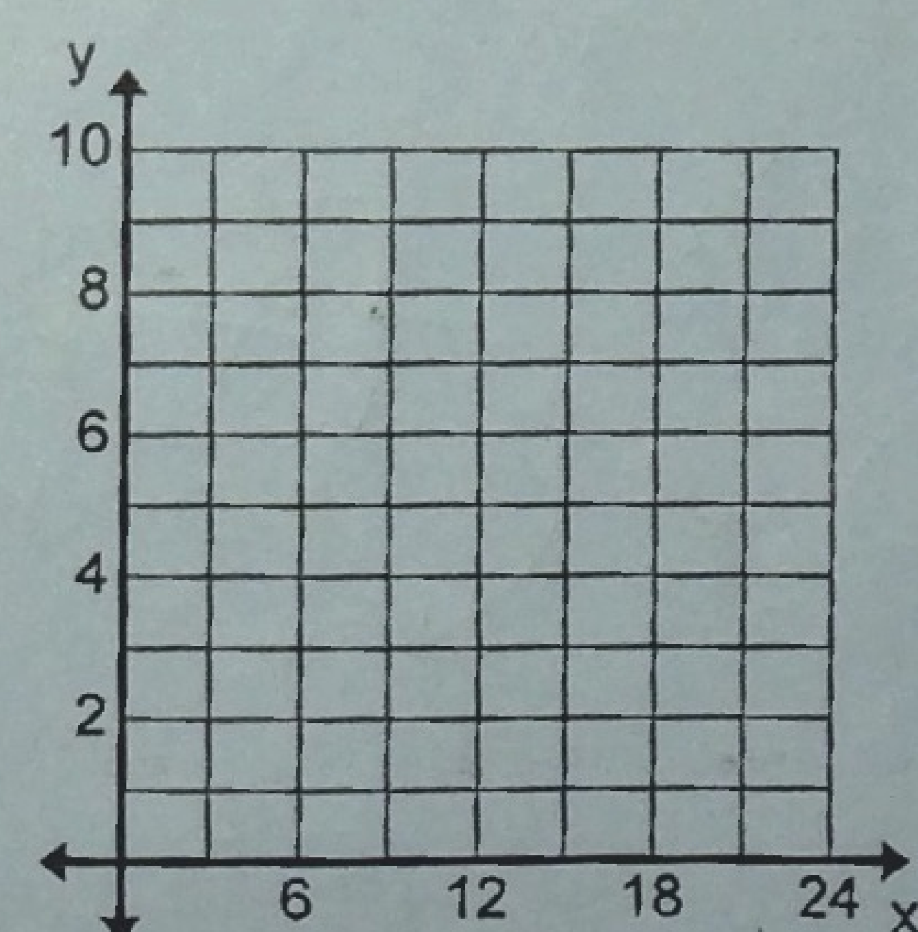
1. y1 $x = t^2 + 5$
y2 $y = \frac{t}{2} + 4$
interval $-4 \le t \le 4$

t	x (y1)	y (y2)	t	x	y
-4	21	2	1	6	4.5
-3	14	2.5	2	9	5
-2	9	3	3	14	5.5
-1	6	3.5	4	21	6
0	5	4			



2. $x = 3t$
$y = \sqrt{t} + 6$
interval $0 \le t \le 8$

t	x	y	t	x	y
0			5		
1			6		
2			7		
3			8		
4					



Converting from Parametric to Rectangular Form

(easy)

1. pick an equation, solve for $\boxed{\text{"}t\text{"}}$
2. Substitute "t" into the other equation
3. Simplify (sometimes solve for "y")

1. $x = -3t$
   $y = t^2 + 2$

$x = -3t$

$\boxed{t = \frac{x}{-3}}$

$y = \left(\frac{x}{-3}\right)^2 + 2$

$\boxed{y = \frac{x^2}{9} + 2}$

2. $x = \sqrt{t} + 4$
   $y = 4t + 3$

$\left(\frac{y-3}{4}\right) = t$

solve for "y".

$x = \sqrt{\frac{y-3}{4}} + 4$

$x = \frac{\sqrt{y-3}}{2} + 4$

$(x-4) = \frac{\sqrt{y-3}}{2}$

$(2(x-4))^2 = (\sqrt{y-3})^2$

$\boxed{3 + 4(x-4)^2 = y}$

3. $x = t^2 - 5$
   $y = 4t$

$\frac{y}{4} = t$

$x = \left(\frac{y}{4}\right)^2 - 5$

$x = \frac{1}{16}y^2 - 5$ OR $x = \frac{y^2}{16} - 5$

$\boxed{\cos^2\theta + \sin^2\theta = 1}$

1) Solve Both equations for "trig part"
2) Substitution - into pyth. idenity
3) Simplify

$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$1 + \tan^2\theta = \sec^2\theta$

4. $x = 2\boxed{\cos T} - 1$
   $y = 2\boxed{\sin T} + 3$

$\left(\frac{x+1}{2}\right) = \cos T$

$\left(\frac{y-3}{2}\right) = \sin T$

$\cos^2\theta + \sin^2\theta = 1$

$\frac{(x+1)^2}{4} + \frac{(y-3)^2}{4} = 1$

$\boxed{(x+1)^2 + (y-3)^2 = 4}$

C: (−1, +3) R: 2

5. $x = 5\cos T - 3$
   $y = 4\sin T - 5$

$\cos t = \frac{x+3}{5}$

$\sin T = \frac{y+5}{4}$

$\frac{(x+3)^2}{25} + \frac{(y+5)^2}{16} = 1$

7. $x = 2(T-2)^2 - 1$
   $y = T$

$x = 2(y-2)^2 - 1$

$x = T$
$y = $ equat

8. $x = 3\sec T + 1$
   $y = 4\boxed{\tan T} - 5$

$\frac{x-1}{3} = \sec T$

$\frac{y+5}{4} = \tan T$

$\sec^2\theta - \tan^2\theta = 1$

$\boxed{\frac{(x-1)^2}{9} - \frac{(y+5)^2}{16} = 1}$

9. $x = 2\tan T + 4$
   $y = 6\sec T - 5$

$\frac{(y+5)^2}{36} - \frac{(x-4)^2}{4} = 1$

Using the CALCULATOR TO GRAPH

$x(t) = 2-t$ AND $y(t) = 2t$ [-3, 3]

BY CALCULATOR:

1. Go to "MODE" and change to parametric mode – "PAR"
2. Go to "y=" and put in $x(t)$ and $y(t)$.
3. Got to WINDOW and looking at your chart above, let's set our window.

T min = -3   T max = 3   T step = .01

X min = -10   X max = 10   X step = 1

Y min = -10   Y max = 10   Y step = 1

4. Now graph and check your graph against the calculator graph.

t	-3	-2	-1	0	1	2	3
x	5	4	3	2	1	0	-1
y	-6	-4	-2	0	2	4	6

Endpoints: S(5,-6) E(-1,6)   Domain: [-1, 5]

Range: [-6, 6]   Function? Why or why not? yes pass VLT

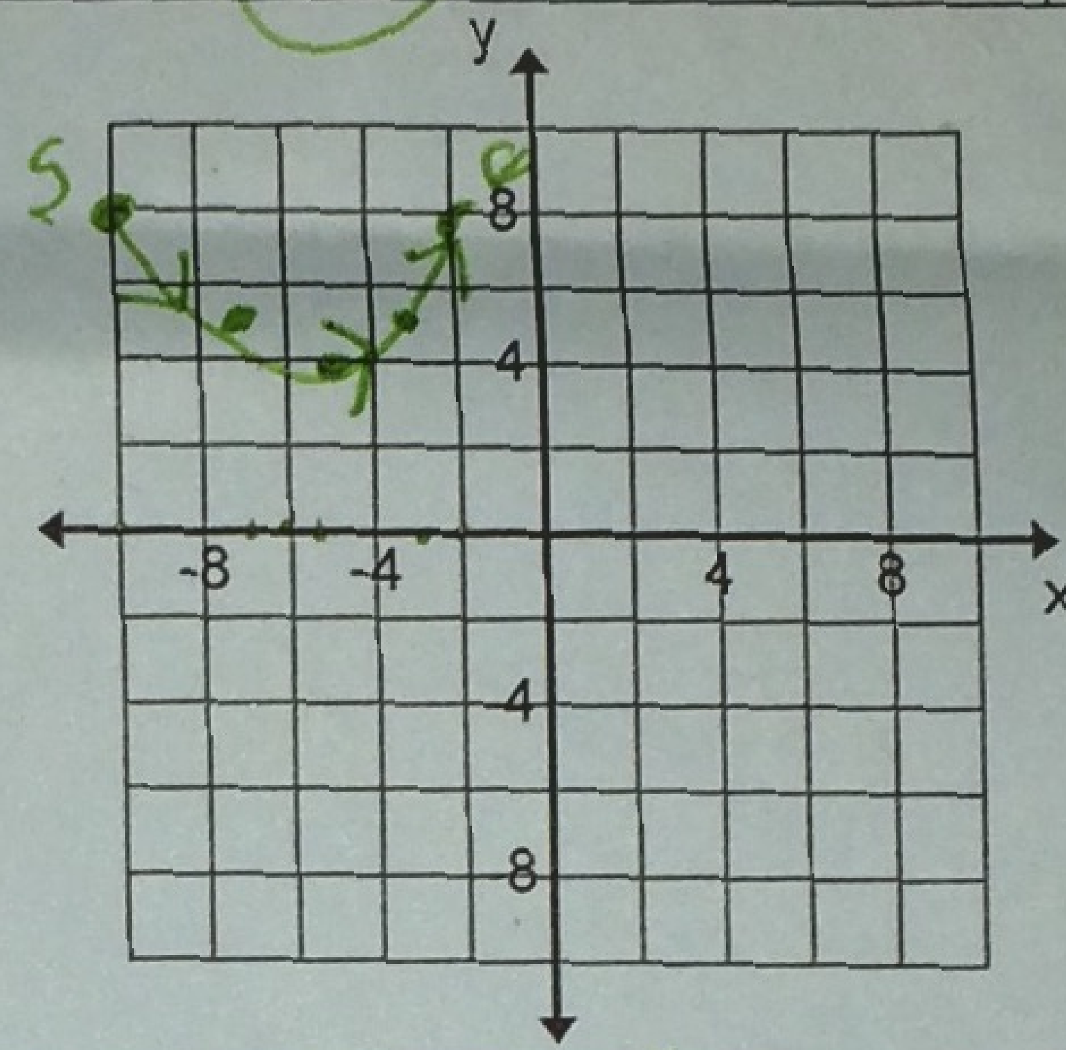
Eliminate the parameters.

$x = 2-t$ $y = 2t$ $y = 2(2-x)$

$\boxed{t = 2-x}$ $\boxed{y = 4-2x}$

10. $x = 2t-5$ AND $y = t^2+4$ $-2 \le t \le 2$

t	-2	-1	0	1	2
x	-9	-7	-5	-3	-1
y	8	5	4	5	8



Endpoints: (-9,8) (-1,8)   Domain: [-9, -1]

Range: [4, 8]   Function? Why or why not? yes Pass VLT

Eliminate the parameters.

$\frac{x+5}{2} = t$ $\boxed{y = \frac{(x+5)^2}{4} + 4}$

11. Chris can sprint at 28 feet per second and Jason sprints at 22 feet per second. Chris gives Jason 30-foot head start.

Jason
Chris   Distance   300 ft

A) Write a pair of parametric equations to represent EACH runner. Remember, $d = rt$.

CHRIS: $x_1 =$ 28t $y_1 =$ 1

JASON: $x_2 =$ 22t+30 $y_2 =$ 3

B) Find a viewing window to simulate a 100-yard dash. WATCH YOUR UNITS.

time [0, 15]

C) Who is ahead after 3 seconds? Who is ahead after 5 seconds? Who wins the race? What was the winner's time?

28t = 300

t = 10.7

t	3 seconds	5 seconds	10.71
Chris	84	140	
Jason	96	140	

12. Luna and Ruby are competing in a 100-meter dash. When the starter gun fires, Luna runs 8.0 meters per Second after a 0.1 second delay and Ruby runs 8.1 meters per second after a 0.2 second delay.

A) Write a pair of parametric equations to represent EACH runner. Remember, $d = rt$.

R
L
S   100

LUNA: $x_1 =$ 8(t+.1) $y_1 = 2$

RUBY: $x_2 =$ 8.1(t+.2) $y_2 = 4$

D = r · t

100 = 8(t+.1)

B) Find a viewing window to simulate a 100-yard dash. WATCH YOUR UNITS.

C) Who is ahead after 3 seconds? Who is ahead after 5 seconds? Who wins the race? What was the winner's time?

t	3 seconds	5 seconds	
Luna	24.8	40.8	
Ruby	25.92	42.12	

If the women ran the 200 meter dash instead of the 100 meter, who would win? Explain your answer.