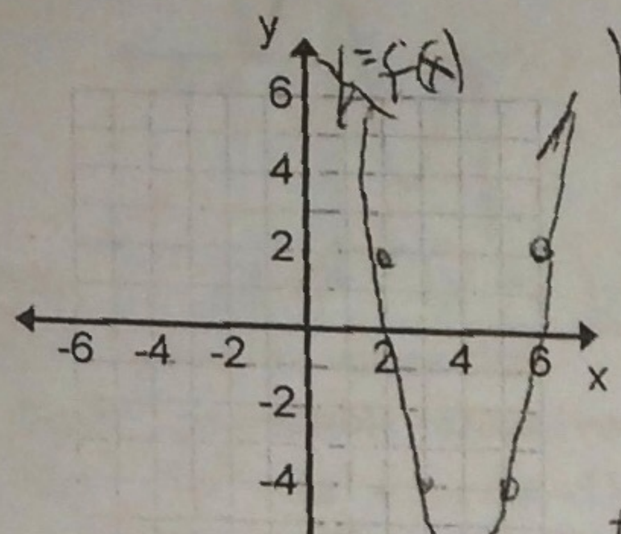
Pre-Calculus TEST 5.1 Review Name KEY per__

Non-Calculator Portion:

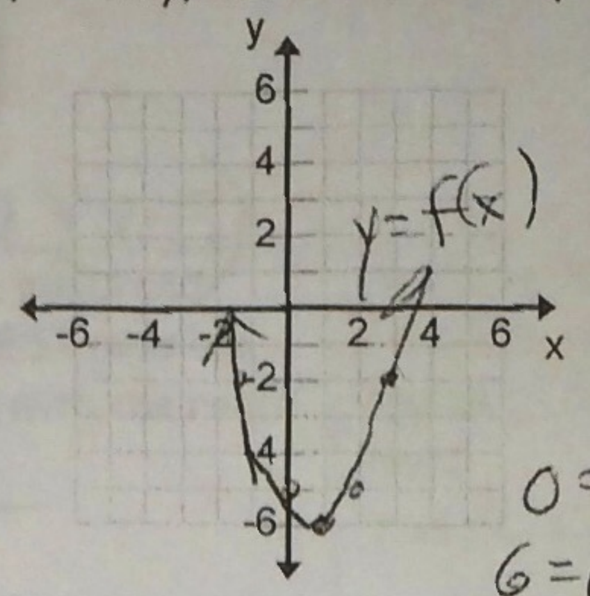
1. Graph $f(x)=2(x-4)^2-6$. Identify the vertex, and the zeros.



VERTEX (4, -6)

ZEROS:
$0=2(x-4)^2-6$
$6=2(x-4)^2$
$3=(x-4)^2$
$\pm\sqrt{3}=x-4$
$x=4\pm\sqrt{3}$ $(4\pm\sqrt{3},0)$

2. Graph $f(x)=x^2-2x-5$. Complete the square to rewrite it in standard form. Identify the vertex, axis of symmetry, and the x-intercepts.



$f(x)=(x^2-2x+1)-5-1$
$f(x)=(x-1)^2-6$
$V(1,-6)$
AXIS OF SYM $x=1$
X INTERCEPTS
$0=(x-1)^2-6$
$6=(x-1)^2$
$\pm\sqrt{6}=x-1$
$(1\pm\sqrt{6},0)$

3. Given $f(x)=3x^2-12x+17$, write in vertex form, identify the vertex, and the x-intercepts.

$a=3$ $b=-12$ $c=17$
$x=-\frac{b}{2a}$ VERTEX $(2, f(2))$ $(2,5)$
$x=\frac{-(-12)}{2\cdot3}$
$x=2$ $f(x)=3(x-2)^2+5$
X INTERCEPTS
$0=3x^2-12x+17$
$x=\frac{-(-12)\pm\sqrt{(-12)^2-4\cdot3\cdot17}}{2\cdot3}$
$2\pm i\sqrt{15}$ NO REAL

4. Write the standard form of the equation of a parabola whose vertex is $(-3,-4)$ and that passes through the point $(-1,-6)$.

$y=a(x-h)^2+k$
$y=a(x+3)^2-4$
$-6=a(-1+3)^2-4$
$-6=a\cdot4-4$
$-2=4a$ $a=-1/2$
$y=-\frac{1}{2}(x+3)^2-4$

Determining characteristics about polynomials from graphs and equations.

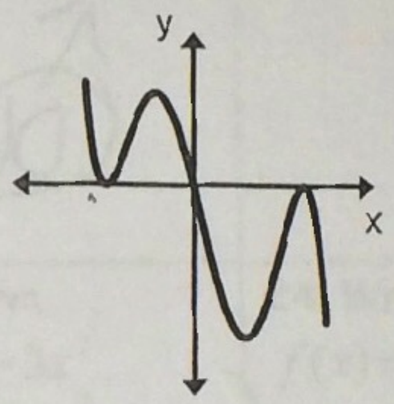
5. $f(x)=-3x^7+4x^5-3x^2+2$

Degree: 7
Possible # Roots: 7
Max # of Rel. Extrema: 6
End Behavior: ↑ ↓
$x\to-\infty$, then $y\to$ $+\infty$
$x\to\infty$, then $y\to$ $-\infty$

6. $f(x)=2x^3(x-4)(x+3)^2$

Degree: 6
Possible # Roots: 6
Max # of Rel. Extrema: 5
End Behavior: ↑ ↑
$x\to-\infty$, then $y\to$ $+\infty$
$x\to\infty$, then $y\to$ $+\infty$

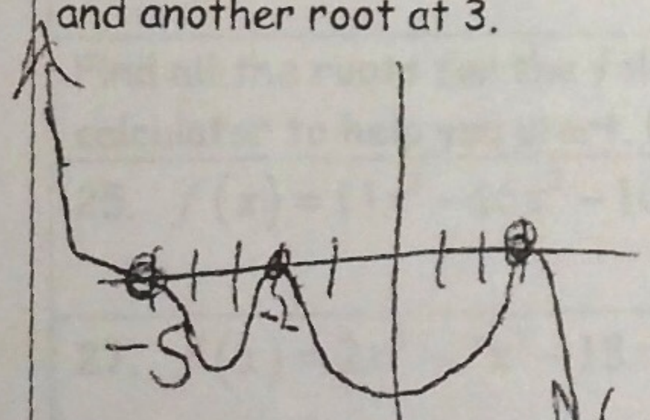
7.



Degree: 5
Actual # Roots: 3
Actual # of Rel. Extrema: 4
End Behavior: ↑ ↓
$x\to-\infty$, then $y\to$ $+\infty$
$x\to\infty$, then $y\to$ $-\infty$

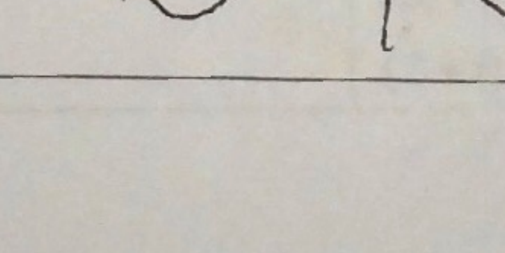
Sketch one possible graph given the following...

8. Sketch a possible graph and write the function in factored form for a polynomial with a degree of 7 and a negative leading coefficient. There are 3 roots, one at -5 with a multiplicity of 3, one at -2 with a multiplicity of 2, and another root at 3.

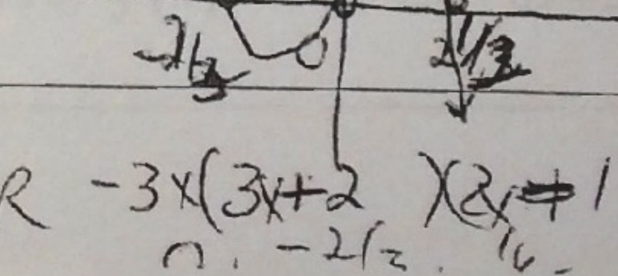


$f(x)=-(x+5)^3(x+2)^2(x-3)$

9. $f(x)=-(x-2)^3(x+3)$



10. $f(x)=(x+4)^2(2x-1)(x+3)$



11. $f(x)=x^4-17x^2+16$

$f(x)=(x^2-16)(x^2-1)$
$f(x)=(x+4)(x-4)(x+1)(x-1)$

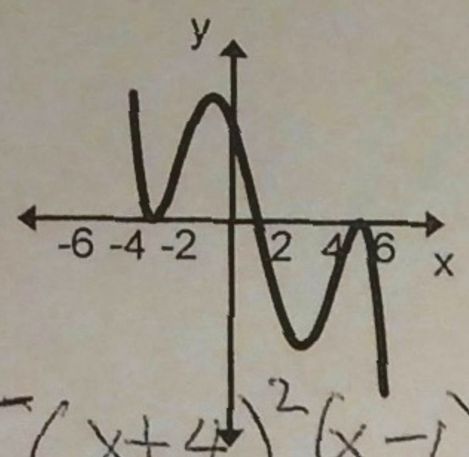
12. $f(x)=-18x^3-3x^2+6x$

$f(x)=-3x(6x^2+x-2)$
$a=6$ $b=-1$ $c=-2$
$x=0$
$\frac{-1\pm\sqrt{(-1)^2-4\cdot6\cdot-2}}{2\cdot6}$

OR $-3x(3x+2)(2x-1)$

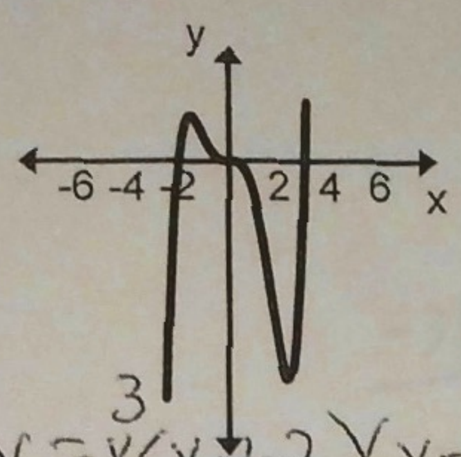
Write equations for the graphs shown here.

13.



$f(x) = -(x+4)^2(x-1)(x-5)^2$

14.



$y = x(x+2)(x-3)$

15. Cross out the values that CANNOT be possible rational roots.

$f(x) = 5x^6 + 3x^5 + \ldots + 6x - 24$

~~5~~	1/5	~~1/12~~	-6
-3/5	12/5	-1	~~120~~

$q: \pm 1, \pm 5$ $p: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

16. Cross out the values that CANNOT be possible rational roots.

$f(x) = 36x^7 + 3x^5 + \ldots + 6x - 15$

-5	~~1/5~~	1/12	~~6~~
~~-3/5~~	3/4	-1	~~36~~

17. Write all conjugate roots and determine the SMALLEST possible degree of a polynomial with the roots:

$2-\sqrt{3}, 4, 0, 2+\sqrt{3}$

4

18. Write all conjugate roots and determine the SMALLEST possible degree of a polynomial with the roots:

$7-i\sqrt{3}, 4+i, 3/4, -\sqrt{3}, 7+i\sqrt{3}, 4-i, +\sqrt{3}$

7

Calculator Portion:

19. Divide using synthetic division:

$(3x^4 + 3x^2 - 2x - 12) \div (x-2)$

2 | 3 0 3 -2 -12
6 12 30 56
3 6 15 28 | 44

QUOTIENT $3x^3 + 6x^2 + 15x + 28$

REMAINDER 44

20. Determine if $(x-4)$ is a factor of $6x^3 - 35x^2 - 79x + 140$.

4 | 6 -35 -79 140
24 -44 -
6 -11 -123 |

NO

21. Determine if $(-\sqrt{3})$ is a root of $3x^3 - 4x^2 - 9x + 12$

$-\sqrt{3}$ | 3 -4 -9 12
$-3\sqrt{3}$ $4\sqrt{3}+9$ -12
3 $-4-3\sqrt{3}$ $4\sqrt{3}$ | 0

YES

22. Find k such that $x+3$ is a factor of $f(x) = x^4 + 4x^3 + kx^2 - 5x - 6$.

-3 | 1 4 K -5 -6
-3 -3 $-3k+9$ $9k-12$
1 1 $k-3$ $-3k+4$ | $9k-18$

$9k - 18 = 0$

$k = 2$

23. Write in factored form.

$f(x) = 2x^5 - 3x^4 - 8x^3 - 3x^2$

$f(x) = x^2(2x^3 - 3x^2 - 8x - 3)$

-1 | 2 -3 -8 -3
-2 5 3
2 -5 -3 | 0

$f(x) = x^2(x+1)(2x^2 - 5x - 3)$

$= x^2(x+1)(2x+1)(x-3)$

$=$

24. Write in factored form.

$f(x) = x^3 + 7x^2 - 54x + 72$

$x = 2$ $x = 3$

2 | 1 7 -54 72
2 18 -72
1 9 -36 | 0

$f(x) = (x-2)(x^2 + 9x - 3$

$f(x) = (x-2)(x+12)(x-$

Find all the roots for the following polynomials. Give EXACT VALUES. Show your work. You may use your calculator to help you start. DO ON YOUR OWN PAPER!!

25. $f(x) = 11x^3 - 16x^2 - 100x - 48$

26. $f(x) = x^6 + 8x^5 + 19x^4 + 8x^3 - 9x^2 - 27$

27. $f(x) = 2x^4 - 7x^3 - 18x^2 + 49x + 28$

28. $f(x) = 2x^3 - 25x^2 + 100x - 125$

29. $f(x) = 2x^3 - 23x^2 + 66x - 54$

30. Find all the roots of $f(x) = 2x^3 - 9x^2 + 6x + 5$ if $(2+i)$ is a root.

25. $f(x)=11x^3-16x^2-100y-48$

$x=-2 \quad x=4$

$-2 \mid 11 \;-16 \;-100 \;-48$
$\quad\quad -22 \;\; 76 \;\; 48$
$\quad 11 \;-38 \;-24 \mid 0$

$4 \mid 11 \;-38 \;-24$
$\quad\quad 44 \;\; 24$
$\quad 11 \;\; 6 \mid 0$

$11x+6=0$

$x=-6/11$

$-2, 4, -6/11$

26. $f(x)=x^6+8x^5+19x^4+8x^3$
$-9x^2-27$

$x=-3, \; x=1$

(TRIPLE ROOT)

$-3 \mid 1 \;\; 8 \;\; 19 \;\; 8 \;\; -9 \;\; 0 \;\; -27$
$\quad\quad -3 \;\; -15 \;\; -12 \;\; 12 \;\; -9 \;\; 27$
$\quad 1 \;\; 5 \;\; 4 \;\; -4 \;\; 3 \;\; -9 \mid 0$

$-3 \mid 1 \;\; 5 \;\; 4 \;\; -4 \;\; 3 \;\; -9$
$\quad\quad -3 \;\; -6 \;\; 6 \;\; -6 \;\; 9$
$\quad 1 \;\; 2 \;\; -2 \;\; 2 \;\; -3 \mid 0$

$-3 \mid 1 \;\; 2 \;\; -2 \;\; 2 \;\; -3$
$\quad\quad -3 \;\; 3 \;\; -3 \;\; 3$
$\quad 1 \;\; -1 \;\; 1 \;\; -1 \mid 0$

$1 \mid 1 \;\; -1 \;\; 1 \;\; -1$
$\quad\quad 1 \;\; 0 \;\; 1$
$\quad 1 \;\; 0 \;\; 1 \;\; 0 \mid 0$

$1x^2+1=0$

$x^2=-1$

$x=\pm i$

$-3, 1, \pm i$

27. $f(x)=2x^4-7x^3-18x^2+49x+28$

$x=4$

$4 \mid 2 \;\; -7 \;\; -18 \;\; 49 \;\; 28$
$\quad\quad 8 \;\; 4 \;\; -56 \;\; -28$
$\quad 2 \;\; 1 \;\; -14 \;\; -7 \mid 0$

$2x^3+x^2-14x-7$

POSSIBLE RATIONAL ROOTS

$p: \pm 1, \pm 2 \qquad p: \pm 1, \pm 7$

$q: \pm 1 \qquad q: \pm 1, \pm 2$

$\frac{p}{q}: \frac{\pm 1}{\pm 1} \; \frac{\pm 7}{\pm 1} \; \frac{\pm 1}{\pm 2} \; \frac{\pm 7}{\pm 2}$

$-\frac{1}{2} \mid 2 \;\; 1 \;\; -14 \;\; -7$
$\quad\quad -1 \;\; 0 \;\; 7$
$\quad 2 \;\; 0 \;\; -14 \mid 0$

$2x^2-14=0$

$2x^2=14$

$x^2=7$

$x=\pm\sqrt{7}$

$4, -\frac{1}{2}, \pm\sqrt{7}$

28. $f(x)=2x^3-25x^2+100x-125$

$x=5$ DOUBLE ROOT

$5 \mid 2 \;\; -25 \;\; 100 \;\; -125$
$\quad\quad 10 \;\; -75 \;\; 125$
$\quad 2 \;\; -15 \;\; 25 \mid 0$

$5 \mid 2 \;\; -15 \;\; 25$
$\quad\quad 10 \;\; -25$
$\quad 2 \;\; -5 \mid 0$

$2x-5=0$

$x=5/2$

$5, 5/2$

29. $f(x) = 2x^3 - 23x^2 + 66x - 54$

$q: \pm 1, \pm 2$ $\quad p: \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm 27, \pm 54$

$3/2$	2	−23	66	−54
		3	−30	54
	2	−20	36	0

$2x^2 - 20x + 36 = 0$

$2(x^2 - 10x + 18) = 0$

QUAD FORMULA

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4 \cdot 1 \cdot 18}}{2}$$

$$x = \frac{10 \pm \sqrt{28}}{2}$$

$$x = \frac{10 \pm 2\sqrt{7}}{2}$$

$$x = 5 \pm \sqrt{7}$$

$3/2, 5 \pm \sqrt{7}$

30. $f(x) = 2x^3 - 9x^2 + 6x + 5$

$2+i$	2	−9	6	5
		$4+2i$	$-5+i$	-5
	2	$-5+2i$	$1+i$	0

$2-i$	2	$-3+2i$	$-2+i$
		$4-2i$	$2-i$
	2	1	0

$2x + 1 = 0$

$x = -1/2$

$2 \pm i, -1/2$