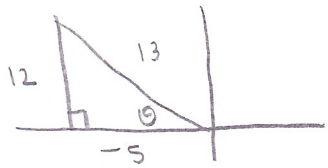
Pre-Calculus Notes

Name: Key

Section 5.5 - Double Angles

Refer to your formula sheet for the Double Angle Formulas.

Example 1: Use double-angle identities to find the exact value. GIVEN: $\cos\theta = -\frac{5}{13}$ and $\frac{\pi}{2} < \theta < \pi$.



$\sin\theta = \frac{12}{13}$

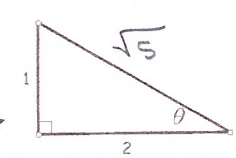
$\cos\theta = \frac{-5}{13}$

$\tan\theta = \frac{12}{-5}$

A. $\sin 2\theta$	B. $\cos 2\theta$	C. $\tan 2\theta$
$= 2\sin\theta\cos\theta$ $= 2\left(\frac{12}{13}\right)\left(\frac{-5}{13}\right)$ $= \boxed{\frac{-120}{169}}$	$= (\cos\theta)^2 - (\sin\theta)^2$ $= \left(\frac{-5}{13}\right)^2 - \left(\frac{12}{13}\right)^2$ $= \boxed{\frac{-119}{169}}$	$= \frac{2\tan\theta}{1-(\tan\theta)^2}$ $= \frac{2\left(\frac{12}{-5}\right)}{\left(1-\left(\frac{-12}{5}\right)^2\right)}$ $= \boxed{\frac{120}{119}}$

Would I want to use the double-angle identity in order to find the EXACT VALUE of $\sin 120^\circ$? **EXPLAIN.**

no! waste of time! reference angle for 120° is 60° and it is in 2nd quadrant, so it equals $\boxed{\frac{\sqrt{3}}{2}}$



Example 2: Use double-angle identities to find the exact value. GIVEN:

1. $\sin\theta$ $\quad \frac{1}{\sqrt{5}} = \boxed{\frac{\sqrt{5}}{5}}$	2. $\cos\theta$ $\quad \frac{2}{\sqrt{5}} = \boxed{\frac{2\sqrt{5}}{5}}$	3. $\tan\theta$ $\quad \boxed{\frac{1}{2}}$
4. $\sin 2\theta$ $= 2\sin\theta\cos\theta$ $= 2\left(\frac{\sqrt{5}}{5}\right)\left(\frac{2\sqrt{5}}{5}\right)$ $= \boxed{\frac{4}{5}}$	5. $\cos 2\theta$ $= (\cos\theta)^2 - (\sin\theta)^2$ $= \left(\frac{2\sqrt{5}}{5}\right)^2 - \left(\frac{\sqrt{5}}{5}\right)^2$ $= \boxed{\frac{3}{5}}$	6. $\tan 2\theta$ $= \frac{2\tan\theta}{1-(\tan\theta)^2}$ $= \frac{2\left(\frac{1}{2}\right)}{\left(1-\left(\frac{1}{2}\right)^2\right)} = \boxed{\frac{4}{3}}$
7. $\csc 2\theta = \frac{1}{\sin 2\theta}$ $= \boxed{\frac{5}{4}}$	8. $\sec 2\theta = \frac{1}{\cos 2\theta}$ $= \boxed{\frac{5}{3}}$	9. $\cot 2\theta = \frac{1}{\tan 2\theta}$ $= \boxed{\frac{3}{4}}$

Now, use your *double angle* formulas to help you solve the following equations.

Example 3: $2\cos x + \sin(2x) = 0$

$2\cos x + 2\sin x \cos x = 0$

$2\cos x(1 + \sin x) = 0$

$* \, n \in \mathbb{Z}$

$\frac{\pi}{2}$, $\frac{3\pi}{2}$

$2\cos x = 0$ OR $1 + \sin x = 0$

$\cos x = 0$ $\sin x = -\frac{1}{2}$

S | A
T | C

ref $\angle \frac{\pi}{6}$

$\boxed{x = \frac{\pi}{2} + \pi n \text{ OR } \frac{7\pi}{6} + 2\pi n \text{ OR } \frac{11\pi}{6} + 2\pi n}$

Example 4: $\cos(2x) - \cos x = 0$

$2\cos^2 x - 1 - \cos x = 0$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0$ $\cos x = 1$ $* \, n \in \mathbb{Z}$

S | A
T | C

ref $\angle \frac{\pi}{3}$

$\cos x = -\frac{1}{2}$

$\boxed{x = \frac{2\pi}{3} + 2\pi n, \ \frac{4\pi}{3} + 2\pi n, \text{ OR } 0 + 2\pi n}$

Example 5:
Ignoring air resistance, the range of a projectile fired at an angle θ with the horizontal and with an initial velocity of v_0 ft per second is given by: $r = \frac{1}{16}v_0^2 \sin\theta\cos\theta$ where r is the horizontal distance (in feet) that the projectile will travel. A place kicker for the football team can kick a football from ground level with an initial velocity of 80 feet per second.

(A) Write the projectile motion model in simpler form.

$r = \frac{1}{16}v_0^2 \sin\theta\cos\theta \Rightarrow$ $r = \frac{1}{16} \cdot \frac{1}{2} V_0^2 \cdot (2\sin\theta\cos\theta)$

$\boxed{r = \frac{1}{32} V_0^2 \sin 2\theta}$

(B) At what angle must the player kick the football so that the football travels 200 feet?

$200 = \frac{1}{32}(80)^2 \sin 2\theta$

$90°$

$1 = \sin 2\theta$

$\frac{2\theta}{2} = \frac{90°}{2}$ $\boxed{\theta = 45°}$

(C) For what angle is the horizontal distance the football travels a maximum?

graph $y = \frac{1}{32}(80)^2 \sin 2\theta$ and use calculator to calculate the maximum

$\boxed{\theta = 45°}$