

Pre-Calculus Notes

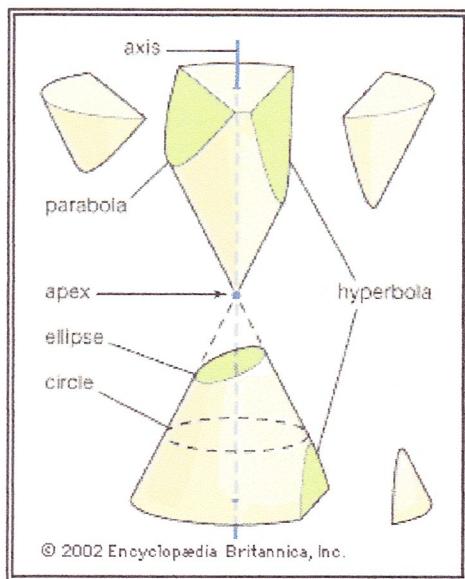
Introduction to Conic Sections

The conic sections result from intersecting a plane with a double cone, as shown in the figure. There are three distinct families of conic sections:

- the ellipse (including the circle)
- the parabola (with one branch)
- the hyperbola (with two branches)

Before we work with these various conic sections, we are going to practice our algebra skills in order to get the equations into standard form. We will also become familiar with identifying the conic section from its equation in standard form.

Name: Key



Standard Form of Conic Sections:

Conic	Equation in Standard Form	Generalization
CIRCLE	$(x-h)^2 + (y-k)^2 = r^2$	sum of x^2 and y^2 w/ same coefficients ex. $3x^2 + 3y^2 = 27$
ELLIPSE	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ OR $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$	sum of x^2 and y^2 w/ DIFFERENT coefficients ex. $5x^2 + 10y^2 = 20$
HYPERBOLA	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ OR $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	difference of x^2 and y^2 or y^2 and x^2 ex. $3x^2 - 3y^2 = 10$ or $4y^2 - x^2 = 12$
PARABOLA	$4c(y-k) = (x-h)^2$ OR $4c(x-h) = (y-k)^2$	only x^2 or only y^2 NOT BOTH ex. $x^2 + y = 5$ or $y^2 - 2y = x$

Example 1: Identify each conic from its equation.

a. $(2x)^2 + 5y^2 - 6y + 16 = y^2$ $4x^2 + 5y^2 - 6y + 16 - y^2 = 0$ $4x^2 + 4y^2 - 6y + 16 = 0$ circle	b. $5x^2 + 3y^2 - 6x + 10 = 8x^2$ $5x^2 + 3y^2 - 6x + 10 - 8x^2 = 0$ $3y^2 - 3x^2 - 6x + 10 = 0$ hyperbola
c. $6x^2 - y^2 + 3x - 2y = 6x^2$ $6x^2 - y^2 + 3x - 2y - 6x^2 = 0$ $-y^2 + 3x - 2y = 0$ parabola	d. $5x^2 + (3y)^2 - 6x + 4y + 2 = 3y^2$ $5x^2 + 9y^2 - 6x + 4y + 2 - 3y^2 = 0$ $5x^2 + 6y^2 - 6x + 4y + 2 = 0$ ellipse

Recall the standard forms for the conic sections:

CIRCLE	$(x-h)^2 + (y-k)^2 = r^2$
ELLIPSE	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ OR $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
HYPERBOLA	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ OR $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
PARABOLA	$4c(y-k) = (x-h)^2$ OR $4c(x-h) = (y-k)^2$

Example 2: Identify each conic section. Then rewrite in standard form by completing the square.

a. $x^2 - 4x + y^2 + 2y + 1 = 0$

$$(x^2 - 4x + \underline{4}) + (y^2 + 2y + \underline{1}) = -1 + \underline{4} + \underline{1}$$

$$(x-2)^2 + (y+1)^2 = 4$$

circle

$x^2 + y^2$
same coefficients

b. $9x^2 + 54x - 4y^2 + 40y - 55 = 0$

$$9x^2 + 54x - 4y^2 + 40y = 55$$

$$9(x^2 + 6x + \underline{9}) - 4(y^2 - 10y + \underline{25}) = 55 + 9(\underline{9}) - 4(\underline{25})$$

$$\frac{9(x+3)^2}{36} - \frac{4(y-5)^2}{36} = 36$$

$$\frac{(x+3)^2}{4} - \frac{(y-5)^2}{9} = 1$$

hyperbola

difference of x^2 and y^2

c. $x^2 - 6x + 8y - 7 = 0$

$$(x^2 - 6x + \underline{9}) = -8y + 7 + \underline{9}$$

$$(x-3)^2 = -8y + 16$$

$$(x-3)^2 = -8(y-2)$$

parabola

only x^2