

Section 10.7 - Polar Coordinates

Cartesian Coordinates:
(x, y)

Polar Coordinates:
(r, θ)

Example 1:
Graph AND label each of the following polar coordinates.

$A(3, 45^\circ)$	$B(3, -45^\circ)$
$C(-3, 45^\circ)$	$D(-3, -45^\circ)$
$E(3, 225^\circ)$	$F(3, 315^\circ)$
$G(-3, 225^\circ)$	$H(3, 0^\circ)$
$I(0, 135^\circ)$	

Example 2: Find three additional polar representations for each of the following points. where $-360^\circ < \theta < 360^\circ$
OR $-2\pi < \theta < 2\pi$

a. $P(5, 40^\circ)$

$(5, 40^\circ - 360^\circ) \rightarrow (5, -320^\circ)$

$(-5, 40^\circ + 180^\circ) \rightarrow (-5, 220^\circ)$

$(-5, 40^\circ - 180^\circ) \rightarrow (-5, -120^\circ)$

b. $P\left(7, -\frac{\pi}{3}\right)$

$\left(7, -\frac{\pi}{3} + 2\pi\right) \rightarrow \left(7, \frac{5\pi}{3}\right)$

$\left(-7, -\frac{\pi}{3} + \pi\right) \rightarrow \left(-7, \frac{2\pi}{3}\right)$

$\left(-7, -\frac{\pi}{3} - \pi\right) \rightarrow \left(-7, -\frac{4\pi}{3}\right)$

Example 3: Graph each polar equation on the Cartesian plane.

a. $r = 4$
circle w/ center $(0, 0)$ and radius of 4

b. $\theta = -2.5$
angle in radians going clockwise -2.14

Recall from last semester:

$\frac{x}{r} = \cos \theta$, when solved for x , becomes $x = r \cos \theta$

$\frac{y}{r} = \sin \theta$, when solved for y , becomes $y = r \sin \theta$

$\frac{y}{x} = \tan \theta$, when solved for θ , becomes $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

And don't forget that $x^2 + y^2 =$ r^2

(x, y) are called rectangular coordinates. (r, θ) are called polar coordinates.

Converting Polar to Rectangular	Converting from Rectangular to Polar
$x = r \cos \theta$ and $y = r \sin \theta$	$\theta = \tan^{-1}(y/x)$ (but double check your quadrant!) and $x^2 + y^2 = r^2$

Example 4: Convert each POLAR point (r, θ) to a RECTANGULAR point (x, y) . SHOW YOUR WORK.

<p>a. $(2, \pi)$</p> $x = r \cos \theta$ $x = 2 \cos \pi$ $x = 2(-1)$ $x = -2$ $y = r \sin \theta$ $y = 2 \sin \pi$ $y = 2 \cdot 0$ $y = 0$ <p>$(-2, 0)$</p>	<p>b. $(\sqrt{3}, \frac{\pi}{6})$</p> $x = \sqrt{3} \cos \frac{\pi}{6}$ $x = \frac{\sqrt{3}}{1} \cdot \frac{\sqrt{3}}{2}$ $x = \frac{3}{2}$ $y = \sqrt{3} \sin \frac{\pi}{6}$ $y = \frac{\sqrt{3}}{1} \cdot \frac{1}{2}$ $y = \frac{\sqrt{3}}{2}$ <p>$(\frac{3}{2}, \frac{\sqrt{3}}{2})$</p>	<p>c. $(-1.3, 2.35)$ radian mode! Use a calculator.</p> $x = -1.3 \cos 2.35$ $x \approx 0.914$ $y = -1.3 \sin 2.35$ $y \approx -0.925$ <p>$(0.914, -0.925)$</p>
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Example 5: Convert each RECTANGULAR point (x, y) to a POLAR point (r, θ) . SHOW YOUR WORK.

<p>a. $(-1, 1) \rightarrow Q II$</p> $x^2 + y^2 = r^2$ $(-1)^2 + 1^2 = r^2$ $1 + 1 = r^2$ $r^2 = 2$ $r = \sqrt{2}$ $\tan \theta = \frac{y}{x}$ $\tan \theta = -1$ $\text{ref } \angle \frac{\pi}{4}$ $\theta = \frac{3\pi}{4}$ <p>$(\sqrt{2}, \frac{3\pi}{4})$</p>	<p>b. $(0, 2)$</p> <p>$(2, \frac{\pi}{2})$</p>	<p>c. $(2, -1) Q IV$</p> $x^2 + y^2 = r^2$ $2^2 + (-1)^2 = r^2$ $5 = r^2$ $r = \sqrt{5}$ $\tan \theta = -\frac{1}{2}$ <p>$(\sqrt{5}, -0.464)$</p>
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Example 6: Convert the rectangular equations to polar form.

<p>a. $x^2 + y^2 = 49$</p> $r^2 = 49$ <p>$r = 7$</p>	<p>$x^2 + y^2 = r^2$</p>	<p>b. $y = x^2$</p> $r \sin \theta = (r \cos \theta)^2$ $r \sin \theta = r^2 \cos^2 \theta$ $\sin \theta = r \cos^2 \theta$ $r = \frac{\sin \theta}{\cos^2 \theta} \rightarrow r = \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta}$ <p>$r = \tan \theta \cdot \sec \theta$</p>	<p>$y = r \sin \theta$ $x = r \cos \theta$</p>
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Example 7: Convert the polar equations to rectangular form.

<p>a. $r^2 = 2$</p> $x^2 + y^2 = 2$	<p>b. $4 \cos \theta = r$</p> $4r \cos \theta = r \cdot r$ $4r \cos \theta = r^2$ $4x = x^2 + y^2$ <p>$x^2 + y^2 - 4x = 0$</p>	<p>$x = r \cos \theta$</p> $r = \frac{1}{\cos \theta}$ $r \cos \theta = 1$ <p>$x = 1$</p>	<p>$\theta = \tan^{-1}(\frac{y}{x})$</p> $\tan^{-1}(\frac{y}{x}) = \frac{\pi}{3}$ $\frac{y}{x} = \tan \frac{\pi}{3}$ $\frac{y}{x} = \sqrt{3}$ <p>$y = \sqrt{3}x$</p>
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