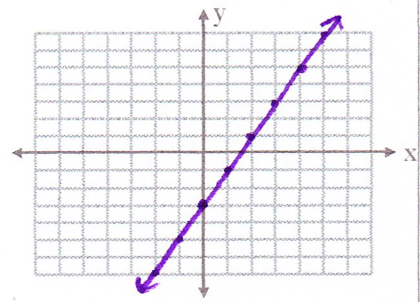
Pre-Calculus Notes   Name________________

Section 12.1 – Intro to Limits

Today we will look at limits, so let's talk about what a limit is...

Ex. 1) $y = 2x - 3$



QUESTION: As the x values get closer to 2, what are the y values getting closer to? 1

NOTATION: $\lim\limits_{x \to 2} 2x - 3 =$ 1

or

$as\ x \to 2,\ \ y \to$ 1

WITH CALCULATOR

Go to table and look at the x values as they approach 2. What are the y values approaching? 1

WITHOUT A CALCULATOR

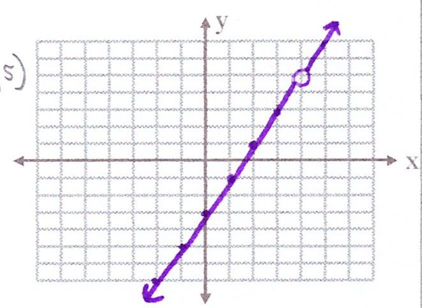
Could we simply substitute the 2 in for x in $y = 2x - 3$? ***Yes.***

$y = 2(2) - 3$
$y = 4 - 3$
$y = 1$

We call this the ***SUBSTITUTION METHOD.***

That was a really simple problem. Let's try something more challenging.

Ex. 2) $y = \dfrac{2x^2 - 11x + 12}{x - 4}$ $y = \dfrac{(2x-3)\cancel{(x-4)}}{\cancel{x-4}}$ $y = 2x - 3$ w/ hole @ (4,5)



QUESTION: As the x values get closer to 4, what are the y values getting closer to? 5

NOTATION: $\lim\limits_{x \to 4} \dfrac{2x^2 - 11x + 12}{x - 4} =$

or

$as\ x \to$ 4, $y \to$ 5

WITH CALCULATOR

Go to table and look at the x values as they approach 4. What are the y values approaching? ?

Do you see a value for y when $x = 4$? no   Why not? would be dividing by zero

Put your calculator on the "ASK" table function and complete the following table:

x	3.9	3.99	3.999	4	4.001	4.01	4.1
y	4.8	4.98	4.998	5	5.002	5.02	5.2

Therefore $\lim\limits_{x \to 4} \dfrac{2x^2 - 11x + 12}{x - 4} =$ 5

WITHOUT A CALCULATOR

Could we simply substitute the 4 in for x in $y = \dfrac{2x^2 - 11x + 12}{x - 4}$? no   Why NOT? denom = 0

So how do we do this problem without a calculator? Factor and reduce!

$y = \dfrac{2x^2 - 11x + 12}{x - 4} = \dfrac{(2x-3)(\cancel{x-4})}{(\cancel{x-4})} =$ $2x - 3$

Now what do we do to get the y value to goes with $x = 4$? $2(4) - 3 = 5$ ☺

We call this the ***CANCELLATION METHOD*** OR ***DIVIDING OUT TECHNIQUE.***

Now let's focus on using the calculator and table for awhile.

Ex. 3) For $f(x) = \dfrac{x^3 - x^2 + x - 1}{x - 1}$, $\lim\limits_{x\to 1} f(x) =$ 2

or $as\ x \to$ 1, $y \to$ 2

WITH CALCULATOR

Go to table and look at the x values as they approach 1. What are the y values approaching? ?

Do you see a value for y when $x = 1$? no Why not? denom would be zero

Put your calculator on the "ASK" table function and complete the following table:

x	0.9	0.99	0.999	1	1.001	1.01	1.1
y	1.81	1.9801	1.998	2	2.002	2.0201	2.21

Ex. 4) For $f(x) = \dfrac{x}{\sqrt{x+1} - 1}$, $\lim\limits_{x\to 0} f(x) =$ 2

WITH CALCULATOR

Go to table and look at the x values as they approach 0. What are the y values approaching? ?

Do you see a value for y when $x = 0$? no Why not? denom would be zero

*

x	~~0.9~~ -.1	~~0.99~~ -.01	~~0.999~~ -.001	~~1~~ 0	~~1~~.001	~~1~~.01	~~1~~.1
y	1.9487	1.995	1.9995	2	2.0005	2.005	2.0488

Ex. 5) $\lim\limits_{x\to -4} \dfrac{\frac{x}{x+2} - 2}{x + 4} =$ .5

WITH CALCULATOR:

x	-4.1	-4.01	-4.001	-4	-3.999	-3.99	-3.9
y	.47619	.49751	.49975	.5	.50025	.50251	.52632

Ex. 6) $\lim\limits_{x\to 0} \dfrac{\sin x}{x} =$ 1

WITH CALCULATOR: Put your mode in RADIANS!!!!

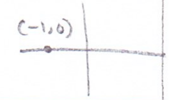
x	-.1	-.01	-.001	0	.001	.01	.1
y	.99833	.99998	.999999	1	.999999	.99998	.99833

Now let's do the following problems using the ***SUBSTITUTION METHOD – NO CALCULATOR!***

Ex. 7) $\lim\limits_{x\to 2} \dfrac{x^2 - x + 1}{x + 1} = \dfrac{2^2 - 2 + 1}{2 + 1} = \dfrac{4 - 2 + 1}{3} = \dfrac{3}{3} = \boxed{1}$

Ex. 8) $\lim\limits_{x \to 0} \dfrac{\sqrt{x+1}+2}{x+3}$ = $\dfrac{\sqrt{0+1}\ +2}{0+3} = \dfrac{3}{3}$ = $\boxed{1}$

Ex. 9) $\lim\limits_{x \to \pi} \dfrac{\tan x}{x}$ = $\dfrac{\tan \pi}{\pi} = \dfrac{0}{\pi}$ = $\boxed{0}$



Ex. 10) $\lim\limits_{x \to 2} e^{3x}$ = $e^{3 \cdot 2}$ = $\boxed{e^{6}}$

Ex. 11) $\lim\limits_{x \to \sqrt{3}} \arcsin\left(\dfrac{x}{2}\right)$ = $\arcsin\left(\dfrac{\sqrt{3}}{2}\right)$ = $\boxed{\dfrac{\pi}{3}}$

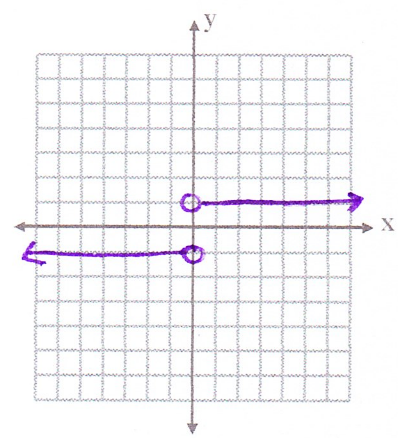
what angle has a sine value of $\frac{\sqrt{3}}{2}$?

Ex. 12) $\lim\limits_{x \to e^2} \ln x$ = $\ln e^2 = 2 \ln e$ = $2 \cdot 1 = \boxed{2}$

Ex. 13) $\lim\limits_{x \to \frac{\pi}{2}} \cos(2x)$ = $\cos\left(2 \cdot \frac{\pi}{2}\right) = \cos \pi$ = $\boxed{-1}$

Do functions always have a limit? Let's look.

Ex. 1) $y = \dfrac{|x|}{x}$



QUESTION: As the x values get closer to 0, what are the y values getting closer to? ?

NOTATION: $\lim\limits_{x \to 0} \dfrac{|x|}{x}$ = DNE

WITH CALCULATOR

As $x \to 0$ from the left side, $y \to$ -1 or always is -1

We call this the ***left- hand limit*** or $\lim\limits_{x \to 0^-} \dfrac{|x|}{x}$ = -1

As $x \to 0$ from the right side, $y \to$ $+1$ or always is $+1$

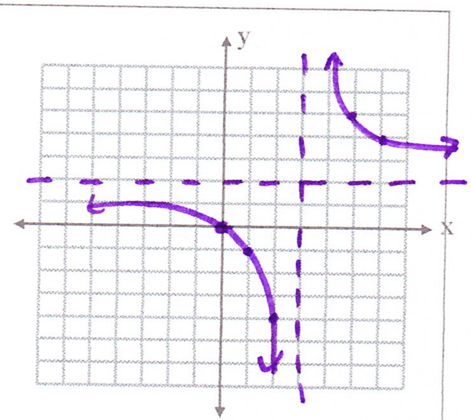
We call this the ***right- hand limit*** or $\lim\limits_{x \to 0^+} \dfrac{|x|}{x}$ = $+1$

Since the y values are approaching different numbers form the left and right, *we do not have a general or overall limit.*

Thus $\lim\limits_{x \to 0} \dfrac{|x|}{x}$ = does not exist

Notice: The left-hand and the right hand limits MUST be the same number in order for the function to have a general limit.

Ex. 2) $y = \dfrac{2x}{x-3}$



QUESTION: As the x values get closer to 3, what are the y values getting closer to? ?

NOTATION: $\lim_{x \to 3} \dfrac{2x}{x-3} =$ DNE

WITH CALCULATOR

As $x \to 3$ from the left side, $y \to$ $-\infty$

X	2.9	2.99	2.999	2.9999	3
y	-58	-598	-5998	-59998	$-\infty$

Thus the ***left- hand limit*** or $\lim_{x \to 3^-} \dfrac{2x}{x-3} =$ $-\infty$

As $x \to 3$ from the right side, $y \to$ $+\infty$

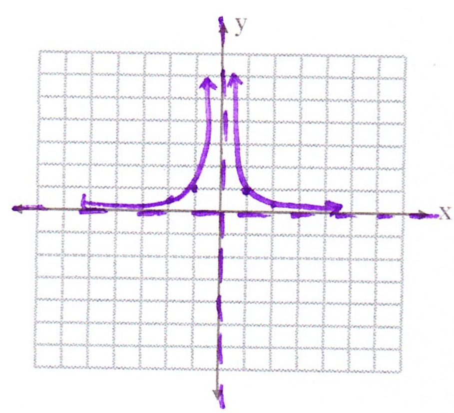
X	3	3.0001	3.001	3.01	3.1
y	$+\infty$	60002	6002	602	62

Thus the ***right- hand limit*** or $\lim_{x \to 3^+} \dfrac{2x}{x-3} =$ $+\infty$

Since the y values are approaching different numbers from the left and right, *we do not have a general or overall limit.*

Thus $\lim_{x \to 3} \dfrac{2x}{x-3} =$ does not exist

Ex. 3) $\lim_{x \to 0} \dfrac{1}{x^2} =$ DNE



WITH CALCULATOR

As $x \to 0$ from the left side, $y \to$ $+\infty$

Thus the ***left- hand limit*** or $\lim_{x \to 0^-} \dfrac{1}{x^2} =$ $+\infty$

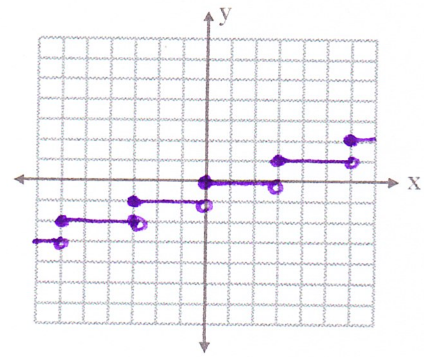
As $x \to 0$ from the right side, $y \to$ $+\infty$

Thus the ***right- hand limit*** or $\lim_{x \to 0^+} \dfrac{1}{x^2} =$ $+\infty$

Thus $\lim_{x \to 0} \dfrac{1}{x^2} =$ does not exist

Please note that some books **DO** allow a limit to be $+\infty$ *or* $-\infty$, but your book **DOES NOT**.

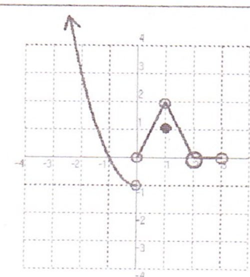
Ex. 4) $f(x)=\left[\!\left[\frac{1}{3}x\right]\!\right]$ - (greatest integer function)



$\lim_{x\to 2^-}\left[\!\left[\frac{1}{3}x\right]\!\right]=$ 0	$\lim_{x\to 3^-}\left[\!\left[\frac{1}{3}x\right]\!\right]=$ 0
$\lim_{x\to 2^+}\left[\!\left[\frac{1}{3}x\right]\!\right]=$ 1	$\lim_{x\to 3^+}\left[\!\left[\frac{1}{3}x\right]\!\right]=$ 1
$\lim_{x\to 2}\left[\!\left[\frac{1}{3}x\right]\!\right]=$ DNE	$\lim_{x\to 3}\left[\!\left[\frac{1}{3}x\right]\!\right]=$ DNE

Now, let's see if we can determine limits by just looking at a graph.

$$y=\begin{cases} x^2-1, & x<0 \\ 2x, & 0<x<1 \\ 1 & x=1 \\ -2x+4, & 1<x<2 \\ 0, & 2<x\le 3 \end{cases}$$



1a. Does $f(-1)$ exist? yes $f(-1)=0$	1b. $\lim_{x\to -1^-} f(x)=$ 0	1c. $\lim_{x\to -1^+} f(x)=$ 0
1d. Does $f(x)$ have a limit at $x=-1$? yes $\lim_{x\to -1} f(x)=0$	1e. Is $f(x)$ continuous at $x=-1$? yes b/c $f(-1)=\lim_{x\to -1} f(x)$	

2a. Does $f(0)$ exist? no → hole	2b. $\lim_{x\to 0^-} f(x)=$ -1	2c. $\lim_{x\to 0^+} f(x)=$ 0
2d. Does $f(x)$ have a limit at $x=0$? no LHL ≠ RHL	2e. Is $f(x)$ continuous at $x=0$? no	

3a. Does $f(1)$ exist? yes $f(1)=1$	3b. $\lim_{x\to 1^-} f(x)=$ 2	3c. $\lim_{x\to 1^+} f(x)=$ 2
3d. Does $f(x)$ have a limit at $x=1$? yes $\lim_{x\to 1} f(x)=2$	3e. Is the limit at $x=1$ the same as the value of $f(x)$ at $x=1$? no	3f. Is $f(x)$ continuous at $x=1$? no

4a. Does $f(2)$ exist? no	4b. $\lim_{x\to 2^-} f(x)=$ 0	4c. $\lim_{x\to 2^+} f(x)=$ 0
4d. Does $f(x)$ have a limit at $x=2$? yes $\lim_{x\to 2} f(x)=0$	4e. Is the limit at $x=1$ the same as the value of $f(x)$ at $x=1$? no	4f. Is $f(x)$ continuous at $x=2$? no