Pre-Calculus Notes

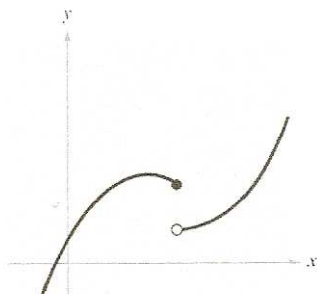**Name:** _____key_____

Section 2.2 – Polynomial Functions of Higher Degree
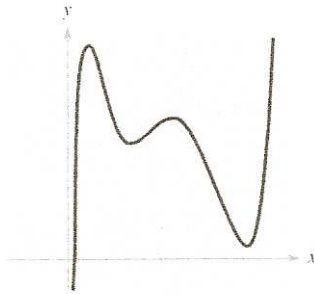
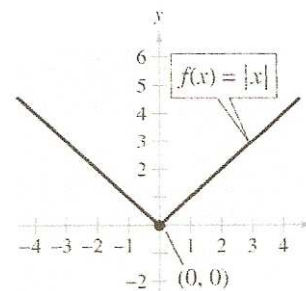**DAY ONE:**   What is true about polynomial functions?
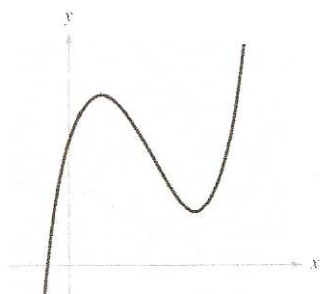
(a) Polynomial functions have continuous graphs.

(b) Functions with graphs that are not continuous are not polynomial functions.

Polynomial functions have graphs with smooth rounded turns.

Graphs of polynomial function cannot have sharp turns.

$f(x) = |x|$

$(0, 0)$

The Leading Coefficient Test/Beginning and Ending Behavior:

Positive lead with odd exponent = ⇓ ⇑   Negative lead with odd exponent = ⇑ ⇓

Positive lead and even exponent = ⇑ ⇑   Negative lead with even exponent = ⇓ ⇓

More Vocabulary:

A *turning point* is a point on a graph such that the value of the function is a relative maximum or relative minimum. This is when the graph switches from increasing to decreasing or from decreasing to increasing. *Relative extrema* consist of relative maximum and relative minimum values of the function. The graph of a polynomial of degree n can have **as many as** $n-1$ relative extrema.

If f is a polynomial function and a is a real number, the following statements are equivalent.
1. $x = a$ is a **zero** of the function f.
2. $x = a$ is a **solution** of the equation $f(x) = 0$.
3. $(x-a)$ is a **factor** of $f(x)$.
4. $(a, 0)$ is an **x-intercept** of the graph of f.

The ___multiplicity___ of a root r is the number of times that $x - r$ is a ___factor___ of $P(x)$. When a real root has an ___even___ multiplicity, the graph of $y = P(x)$ touches the ___x-axis___ but does not cross it. When a real root has ___odd___ multiplicity greater than 1, the graph "___bends___" as it crosses the ___x-axis___. In this class, we will primarily deal with multiplicities of 1, 2, or 3.
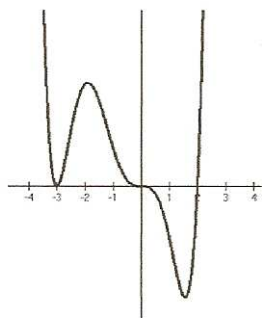
End Behavior: ⇑ ⇑
Degree: even
Leading Coefficient: positive
Number of relative extrema: 3
Zeros and their Muliplicity:
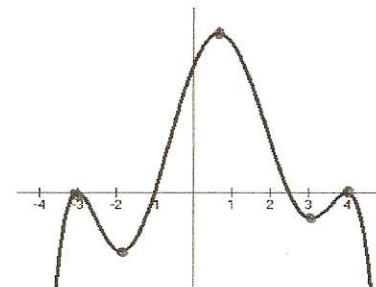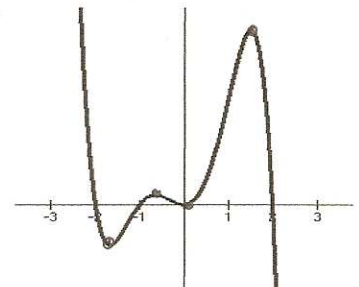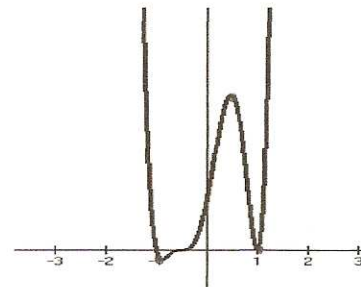-3 w/ multiplicity of 2 (double root)
0 w/ multiplicity of 3 (triple root)
2 w/ multiplicity of 1 (single root)
Possible Function (factored form):
$y = +x^3 (x+3)^2 (x-2)$

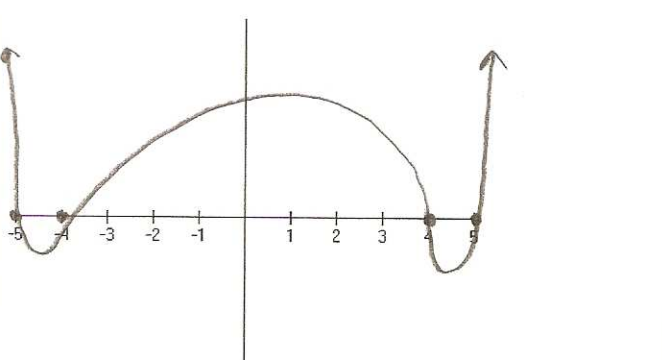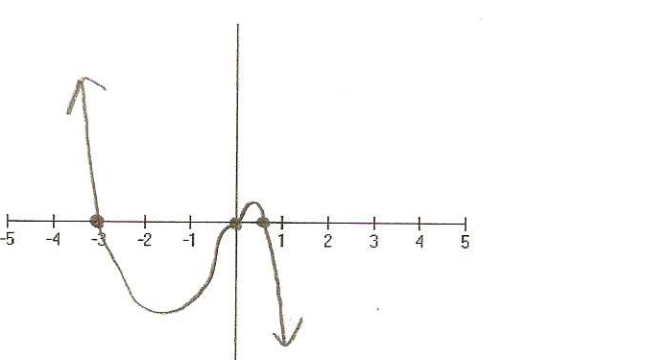**Example 1:** Find the requested information for each graph.

a.	b.	c.

a.

End Behavior: ↓↓

Even or Odd Degree? __even__

+ or - Leading Coeff.: __−__

# of relative extrema: __5__

Zeros and their Muliplicity:

-3   triple
-1   single
$\frac{5}{2}$ single
4   double

Possible Function (factored form):

$$y = -(x+3)^3(2x-5)(x+1)(x+4)^2$$

b.

End Behavior: ↑↓

Even or Odd Degree? __odd__

+ or - Leading Coeff.: __−__

# of relative extrema: __4__

Zeros and their Muliplicity:

-2   single
-1   single
0   double
2   single

Possible Function (factored form):

$$y = -x^2(x+2)(x+1)(x-2)$$

c.

End Behavior: ↑↑

Even or Odd Degree? __even__

+ or - Leading Coeff.: __+__

# of relative extrema: __3__

Zeros and their Muliplicity:

-1   single
$-\frac{1}{2}$ triple
1   double

Possible Function (factored form):

$$y = +(x+1)(2x+1)^3(x-1)$$

(possible)

Example 2: Determine the **MAXIMUM number of relative extrema for each polynomial function**. State the beginning and ending behavior of the curve (⇑⇑, ⇑⇓, ⇓⇓, or ⇓⇑).

a. $f(x) = 3x^5 - 3x^2 + 2x - 1$ + odd

$5 - 1 = \boxed{4}$ $\boxed{↓↑}$

b. $f(x) = -x^3 + 2x - 1$ − odd

$3 - 1 = \boxed{2}$ $\boxed{↑↓}$

Example 3: Factor each polynomial. Then identify its roots and their multiplicity along with the end behavior in order to sketch a graph WITHOUT the calculator.

a. $f(x) = x^4 - 29x^2 + 100$ ↑↑

$f(x) = (x^2 - 25)(x^2 - 4)$

$f(x) = (x-5)(x+5)(x-4)(x+4)$
      5   -5   4   -4

b. $f(x) = -2x^5 - 5x^4 + 3x^3$ ↑↓

$f(x) = -x^3(2x + 5x - 3)$

$f(x) = -x^3(2x - 1)(x + 3)$
      0        $\frac{1}{2}$ -3
   triple

DAY TWO:

Example 4: Use a graphing utility to graph a sketch of the function. Determine each real zero AND the number of relative extrema. Then express the function as a product of linear factors.

a. $f(x) = x^4 + 3x^3 - 4x$	b. $f(x) = -2x^4 - 3x^3 + 9x^2 - x - 3$
# Relative Extrema: ___3___	# Relative Extrema: ___3___
Real Zeroes (and their multiplicity):	Real Zeroes (and their multiplicity):
-2 double 0 single 1 single	-3 single $-\frac{1}{2}$ single 1 double
Factored Form of Function:	Factored Form of Function:
$f(x) = + x(x+2)^2(x-1)$	$f(x) = -(x-1)^2(x+3)(2x+1)$

Example 5: Sketch a graph of the function WITHOUT using a calculator.

a. $y = x(x+3)^3(x-2)^2$
  $+$
  0 -3 triple 2 double

b. $y = -(x+4)^2(2x-1)(x-4)$
  -4 double $\frac{1}{2}$ 4

Example 5: Determine a possible equation for the polynomial pictured in the graph.

a.

$y = + x^2(x+2)^2(2x+1)(x-1)$

$-\frac{1}{2}$ single
0 double
1 single
-2 double

b.

$y = -(x+1)^3(x-2)^2$

-1 triple 2 double

Example 6: Miscellaneous Problems. ☺

a. Find all the zeroes for	b. Sketch the graph of	c. Write an equation with degree
$f(x) = -\frac{1}{4}(x-3)^2(x+2)^3.$	$f(x) = 3x^4 - 4x^3.$ $x^3(3x-4)$	$n=5$ and having 2 as a triple root and 1 as a double root.
3 (double) -2 (triple)	0 triple $\frac{4}{3}$	$y = +(x-2)^3(x-1)^2$ OR $y = -(x-2)^3(x-1)^2$ *End behavior not specified.