Pre-Calculus Notes

Name: key

Section 2.3 – Polynomial and Synthetic Division

REMAINDER THEOREM:

When the polynomial $P(x)$ is divided by $x-a$, the remainder is $P(a)$.

FACTOR THEOREM:

A number a is a solution of the polynomial equation $f(x)=0$ if and only if $x-a$ is a factor of $f(x)$. * To be a factor, the remainder when $f(x)$ is divided by $x-a$ **MUST BE ZERO.**

Example 1: Using the theorems...

a. Find the remainder when $3x^5-2x^3+7x^2-10$ is divided by $x+5$. Show your work by doing long division.

$$3x^4-15x^3+73x^2-358x+1790$$

$$x+5 \overline{)\ 3x^5+0x^4-2x^3+7x^2+0x-10}$$

$$+\ -3x^5-15x^4$$

$$-15x^4-2x^3$$

$$+\ +15x^4+75x^3$$

$$73x^3+7x^2$$

$$+\ -73x^3-365x^2$$

$$-358x^2+0x$$

$$+\ +358x^2+1790x$$

$$1790x-10$$

$$+\ -1790x-8950$$

$$-8960$$

$$\boxed{3x^4-15x^3+73x^2-358x+1790-\frac{8960}{x+5}}$$

b. Is -2 a solution of the equation $x^6+4x^3-2x^2+5x-1=0$? Why or why not?

$(-2)^6+4(-2)^3-2(-2)^2+5(-2)-1=0?$

$64-32-8-10-1=0?$

$13\neq 0$

no b/c when you plug -2 into the equation you get a false statement!

Example 2: Use synthetic division to divide. * use synthetic division when dividing by $x-a$

a. $(2x^3-3x^2+4)\div(x-2)$

2	2	-3	0	4
	↓	4	2	4
	2	1	2	8

$$\boxed{2x^2+x+2+\frac{8}{x-2}}$$

b. $(6x^3-2x^2+5x+1)\div(x+1)$

-1	6	-2	5	1
	↓	-6	8	-13
	6	-8	13	-12

$$\boxed{6x^2-8x+13-\frac{12}{x+1}}$$

c. $(8x^4-16x^3+16x^2-27x+18)\div(2x-3)$

$(8x^4-16x^3+16x^2-27x+18)\div 2(x-\frac{3}{2})$

$\frac{3}{2}$	8	-16	16	-27	18
		12	-6	15	-18
	8	-4	10	-12	0

$\div 2$

4 -2 5 -6

$\boxed{4x^3-2x^2+5x-6}$

d. $(15x^4-x^3-11x^2+17x+9)\div(5x+3)$

$(15x^4-x^3-11x^2+17x+9)\div 5(x+\frac{3}{5})$

$-\frac{3}{5}$	15	-1	-11	17	9
		-9	6	3	-12
	15	-10	-5	20	-3

$\div 5$

3 -2 -1 4

$\boxed{3x^3-2x^2-x+4-\frac{3}{5x+3}}$

Example 3:

a. Use synthetic substitution to evaluate the function.
$f(-2)$ for $f(x)=x^4-3x^3+7x^2-20$

-2	1	-3	7	0	-20
		-2	10	-34	68
	1	-5	17	-34	48

$\boxed{f(-2)=48}$

b. Determine k so that $g(x)=2x^3+5x^2+kx-16$ has $x-2$ as a factor. ← no remainder

2	2	5	k	-16
		4	18	$2k+36$
	2	9	$k+18$	$2k+20$

$2k+20=0$

$2k=-20$

$\boxed{k=-10}$

We will now do p.159-161 #7, 13, 19, 25, 45(a,b), 69, 84 (see below) as classwork. It is due IN CLASS.

7. Use long division to divide $(4x^3-7x^2-11x+5)\div(4x+5)$.

13. Use long division to divide $(6x^3+10x^2+x+8)\div(2x^2+1)$.

19. Use synthetic division to divide $(3x^3-17x^2+15x-25)\div(x-5)$.

25. Use synthetic division to divide $(5x^3-6x^2+8)\div(x-4)$.

45. Use synthetic division to find each function value. $f(x)=4x^3-13x+10$.

   a. $f(1)$

   b. $f(-2)$

69. Simplify the rational expression $\dfrac{4x^3-8x^2+x+3}{2x-3}$ by using long division or synthetic division.

84. Find the constant c such that the denominator will divide evenly into the numerator for the rational expression $\dfrac{x^3+4x^2-3x+c}{x-5}$.