Pre-Calculus Notes Name: Key

Section 2.5 – Zeroes of Polynomial Functions

Rational Zeros (Roots) Theorem:

Let $f(x)=a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$ (where $a_0 \neq 0$) be a polynomial function in standard form that has integral coefficients. THEN if the nonzero rational number $\frac{p}{q}$ in lowest terms is a zero of $p(x)$, p must be a factor of the constant term a_0 AND q must be a factor of the leading coefficient a_n.

Example 1: Determine the **possible** rational zeros of each polynomial.

a. $f(x)=1x^4-7x^3-3x^2+2x+12$

$p = \pm1, \pm2, \pm3, \pm4, \pm6 \pm12$

$q = \pm1$

$\boxed{\frac{p}{q} = \pm1, \pm2, \pm3, \pm4, \pm6, \pm12}$

b. $g(x)=6x^4-3x^3+x^2-10x+15$

$p = \pm1, \pm3, \pm5, \pm15$

$q = \pm1, \pm2, \pm3, \pm6$

$\boxed{\frac{p}{q} = \pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}}$

What kind of polynomial equation(s) can I solve easily? quadratic!

Example 2: Determine the EXACT VALUES of the zeroes of each polynomial. Use your calculator to get started.

a. $p(x)=7x^3+18x^2-97x-60$

from calculator: zero @ 3 and @ −5

3	7	18	−97	−60
	↓	21	117	60
	7	39	20	0

−5	7	39	20
	↓	−35	−20
	7	4	0

$7x+4=0$

$7x=-4$

$x=-\frac{4}{7}$

$\boxed{\left\{-5, -\frac{4}{7}, 3\right\}}$

b. $f(x)=22x^4+65x^3-20x^2-45x+18$

from calc: zeroes @ −1 and −3

−1	22	65	−20	−45	18
	↓	−22	−43	63	−18
	22	43	−63	18	0

−3	22	43	−63	18
	↓	−66	69	−18
	22	−23	6	0

mult = 132
add = −23
−11, −12

$22x^2-23x+6=0$

$(11x-6)(2x-1)=0$

$x=\frac{6}{11}, \frac{1}{2}$

$\boxed{\left\{-3, -1, \frac{1}{2}, \frac{6}{11}\right\}}$

Fundamental Theorem of Algebra:
Every polynomial function of positive degree with complex coefficients has at least one complex zero.

NOTE: The zero may be a real number since ANY real number r can be expressed as the complex number $r+0i$.

Number of Roots Theorem:
If $f(x)$ is a polynomial of degree $n \geq 1$ with complex coefficients, then $f(x)=0$ has exactly n roots, where roots are counted according to their multiplicity.

Conjugate Pair Theorem:
If $f(x)=0$ is a polynomial equation real coefficients, then when $a+bi$ is a root, $a-bi$ is also a root. If $f(x)=0$ is a polynomial equation rational coefficients, then when $m+\sqrt{n}$ is a root, $m-\sqrt{n}$ is also a root.

Example 3: Determine the zeros of each polynomial.

a. $f(x)=x^4-x^3+x^2-3x-6$

from calc: $-1, 2$

$$\begin{array}{r|rrrr|r} -1 & 1 & -1 & 1 & -3 & -6 \\ & & -1 & 2 & -3 & 6 \\ \hline & 1 & -2 & 3 & -6 & 0 \end{array}$$

$$\begin{array}{r|rrr|r} 2 & 1 & -2 & 3 & -6 \\ & & 2 & 0 & 6 \\ \hline & 1 & 0 & 3 & 0 \end{array}$$

$x^2+3=0$

$x^2=-3$

$x=\pm i\sqrt{3}$

$\boxed{\{-1, 2, \pm i\sqrt{3}\}}$

b. $g(x)=x^5+5x^4-8x^3-40x^2$

$g(x) = x^2\underbrace{(x^3+5x^2-8x-40)}_{\text{from calc: } -5}$

0

$$\begin{array}{r|rrr|r} -5 & 1 & 5 & -8 & -40 \\ & & -5 & 0 & 40 \\ \hline & 1 & 0 & -8 & 0 \end{array}$$

$x^2-8=0$

$x^2=8$

$x=\pm 2\sqrt{2}$

$\boxed{\{-5, 0, \pm 2\sqrt{2}\}}$

double root

c. $f(x)=9x^4+131x^3+183x^2+9x-52$

from calc: -1 (double)

sometimes a standard window doesn't cut it!

$$\begin{array}{r|rrrr|r} -1 & 9 & 131 & 183 & 9 & -52 \\ & & -9 & -122 & -61 & 52 \\ \hline & 9 & 122 & 61 & -52 & 0 \end{array}$$

$$\begin{array}{r|rrr|r} -1 & 9 & 122 & 61 & -52 \\ & & -9 & -113 & 52 \\ \hline & 9 & 113 & -52 & 0 \end{array}$$

mult $=-117$
add $=113$
-4 and 117

$9x^2+113x-52=0$ (-117)

$(9x-4)(x+13)=0$

$x=\frac{4}{9}$ $x=-13$

$\boxed{\{-13, -1, \frac{4}{9}\}}$

double root

d. $g(x)=2x^4+11x^3+2x^2-65x-100$

from calc: $-4, \frac{5}{2}$

$$\begin{array}{r|rrrr|r} -4 & 2 & 11 & 2 & -65 & -100 \\ & & -8 & -12 & 40 & 100 \\ \hline & 2 & 3 & -10 & -25 & 0 \end{array}$$

$$\begin{array}{r|rrr|r} \frac{5}{2} & 2 & 3 & -10 & -25 \\ & & 5 & 20 & 25 \\ \hline & 2 & 8 & 10 & 0 \end{array}$$

$2x^2+8x+10=0$

$x^2+4x+5=0$

doesn't factor!

quad form or comp. square

$x^2+4x+\underline{4}=-5+\underline{4}$

$\sqrt{(x+2)^2}=\sqrt{-1}$

$x+2=\pm i$

$x=-2\pm i$

$\boxed{\{-4, \frac{5}{2}, -2\pm i\}}$

Example 4: Determine the remaining zeros of the polynomial given one of the zeros. Explain how you arrived at your answer.

$f(x) = x^4 - 13x^3 + 61x^2 - 127x + 78$; $3+2i$

$3+2i$	1	-13	61	-127	78
		$3+2i$	$-34-14i$	$109+12i$	-78
	1	$-10+2i$	$27-14i$	$-18+12i$	0

$3-2i$	1	$-10+2i$	$27-14i$	$-18+12i$
		$3-2i$	$-21+14i$	$18-12i$
	1	-7	6	0

$x^2 - 7x + 6 = 0$

$(x-6)(x+1) = 0$

$x = 6,\ x = -1$

$\boxed{\{-1, 6, 3 \pm 2i\}}$

Example 5: Use your calculator to answer the following questions about the given polynomial function.

$f(x) = x^4 - 7x^3 - 46x^2 + 14x + 88$

$p = \pm 1, \pm 2, \pm 4, \pm 8, \pm 11, \pm 22, \pm 44, \pm 88$

$q = \pm 1$

Possible Rational Roots: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 11, \pm 22, \pm 44, \pm 88$

Actual Rational Roots: $-4, 11$

# of Real Roots: 4

# of Irrational Roots: 2

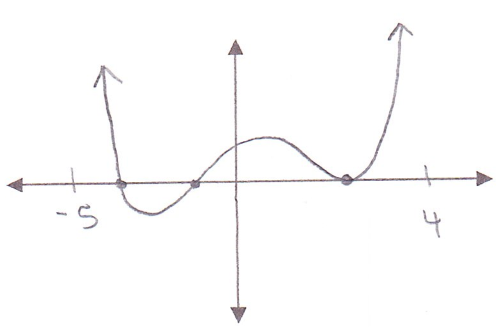
Irrational Roots: ≈ -1.41 & 1.41 (Use the zero feature on your graphing calculator to estimate.)

# of Imaginary Roots: 0

Imaginary Roots: —

Example 6: Sketch a possible graph with the following conditions.

a. A fourth degree polynomial with a positive leading coefficient, two distinct negative real zeros greater than -5, and one positive real zero less than 4 with a multiplicity of 2.



b. A fifth degree polynomial with a negative leading coefficient, two distinct negative real zeros greater than -4 (one with multiplicity 3), and one positive real zero less than 3.