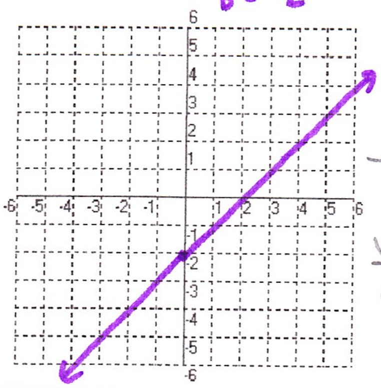
Pre-Calculus NOTES
Section 2.6 Rational Functions

Name: Key

DAY ONE:

Graph the line $y = x - 2$. Is it continuous?
EXPLAIN.

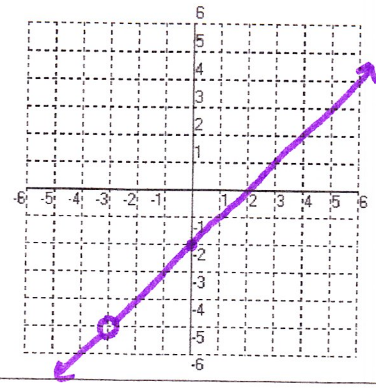
$m = 1$
$b = -2$



continuous
→ can trace w/ pencil w/o lifting it off the paper

NOW graph $y = \frac{(x-2)\cancel{(x+3)}}{\cancel{x+3}}$. Is it continuous?
EXPLAIN.

$x \neq -3$ (can't ÷ by 0)



discontinuous
HOLE @ $(-3, -5)$

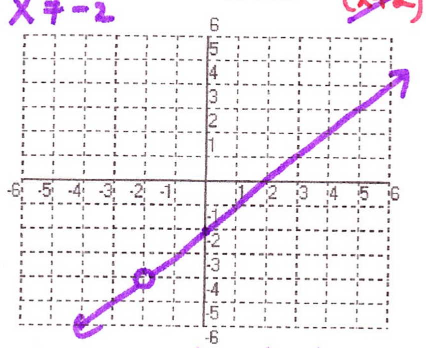
A **rational function** is a fraction whose numerator and denominator are polynomials. The **most important difference** between graphs of rational functions and the graph of polynomials is the presence of **discontinuities** such as **holes** or **asymptotes**. A **hole** is a **point** where the function is **undefined**. An **asymptote** to a graph is a **straight line** that the **graph squeezes up against**, without actually touching.

3 kinds of discontinuity:

POINT (hole)

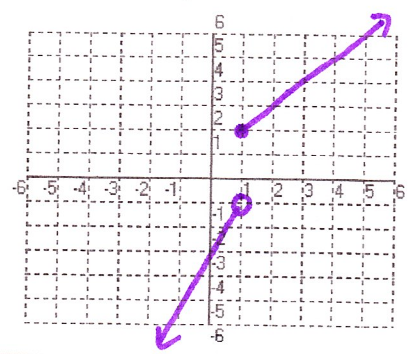
$f(x) = \frac{x^2 - 4}{x+2}$ $\quad \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}}$

$x \neq -2$



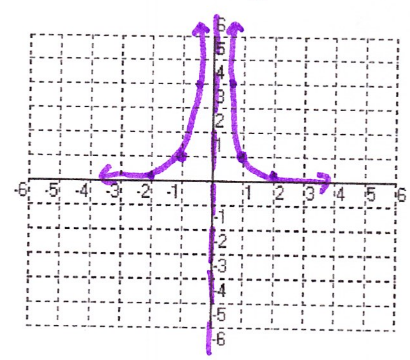
JUMP (gaps)

$f(x) = \begin{cases} x+1, & x \geq 1 \\ 2x-3, & x < 1 \end{cases}$



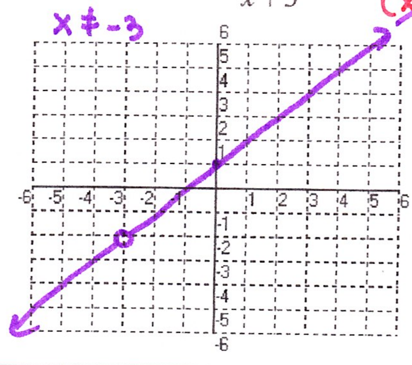
INFINITE (asymptotes)

$f(x) = \frac{1}{x^2}$ $\quad x \neq 0$

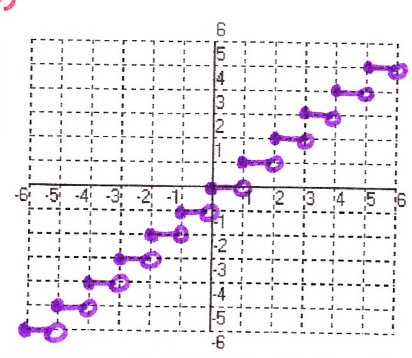


$f(x) = \frac{\cancel{x^2 - 7x + 10}\; x^2 + 4x + 3}{x+3}$ $\quad \frac{\cancel{(x+3)}(x+1)}{\cancel{(x+3)}}$

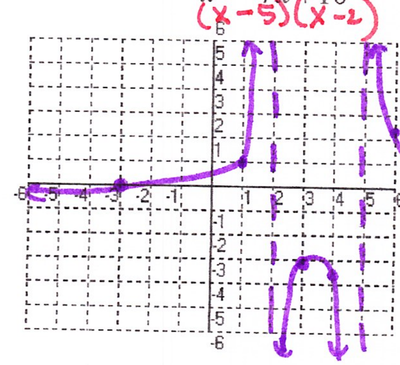
$x \neq -3$



$f(x) = [\![x]\!]$

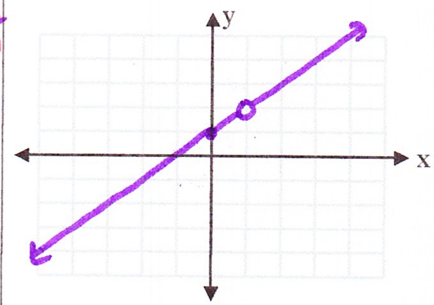


$f(x) = \frac{x+3}{x^2 - 7x + 10}$ $\quad \frac{x+3}{(x-5)(x-2)}$ $\quad x \neq 2, 5$



x	y
-3	0
1	1
3	-3
4	-2.5
6	2.25

Example 1:

a. Determine the place(s) of discontinuity for the function $f(x)=\frac{1}{x-4}$. $x-4\neq 0$ $x\neq 4$ asymptote @ $x=4$	b. Determine the place(s) of discontinuity for the function $g(x)=\frac{x^2-1}{x^2+3x+2}$. $\frac{\cancel{(x+1)}(x-1)}{(x+2)\cancel{(x+1)}}$ hole @ $x=-1$ asymptote @ $x=-2$	c. Graph the function $\frac{x^2-1}{x-1}$. $\frac{\cancel{(x+1)}\cancel{(x-1)}}{\cancel{x-1}}$ $x\neq 1$ 

DAY TWO:

Memorize!

Let $\frac{f(x)}{g(x)}$ be a rational function where $f(x)$ is a polynomial of degree n and $g(x)$ is a polynomial of degree m, then the following asymptote rules apply…

VERTICAL ASYMPOTE:

If $g(a)=0$ and $f(a)\neq 0$, then $x=a$ is a vertical asymptote.

HORIZONTAL ASYMPOTES:

a) If $n<m$, $y=0$ is a horizontal asymptotes.

b) If $n>m$, there is no horizontal asymptote… probably a slanted asymptote.

c) If $n=m$, $y=c$ is horizontal asymptote, where c is the quotient of the leading coefficients.

SLANTED ASYMPTOTES:

The equation of the slant asymptote can be found by dividing and then rewriting the function.

Big Time Warning:

A horizontal or a slant asymptote has NO effect on the graph except at the right and left extremes. In particular, it is NOT TRUE that a graph can never cross its horizontal or slant asymptote. The graph might cross its horizontal or slant asymptote many times toward the "middle" of the graph. Only when the graph makes its "final approach" to the asymptote, at the left or right extremes of the graph, will it squeeze up against the asymptote without ever touching it. (By the way, it IS TRUE that the graph can never touch one of its vertical asymptotes.)

Procedure for Graphing Rational Functions:

1. Find the zeros of the denominator, and use the answer(s) to draw the vertical asymptote(s) with dotted lines or to identify any holes in the graph.
2. Use the procedure given earlier to find and draw the horizontal or slant asymptote, if any.
3. Find the zeros of the numerator, and use them to plot the x-intercept(s). Then draw a short piece of the graph near the x-intercept(s).
4. Decide what the graph looks like near the vertical asymptote(s), and appropriately draw short pieces of the graph as the graph approaches each vertical asymptote from the left and the right.
5. Decide what the graph looks like near the horizontal or slant asymptote, if there is one. The draw short pieces pf the graph at the far left and right extremes as the graph approaches the asymptote.
6. If the graph has no horizontal or slant asymptote. Determine what happens to the graph at the far left and right extremes by substituting a large positive and a large negative number for x. Then draw short pieces of the graph at the far left and far right extremes.
7. Connect the pieces of the graph that you have already drawn (including the x-intercepts), with a smooth curve.

Example 2: Graph each function. State the equation of ALL asymptotes: vertical, horizontal, or slanted. Determine any intercepts.

a. $g(x)=\frac{4x+3}{x+1}$

VA : $x=-1$

HA : $y=4$

x-int : $0=\frac{4x+3}{x+1}$

$0=4x+3$

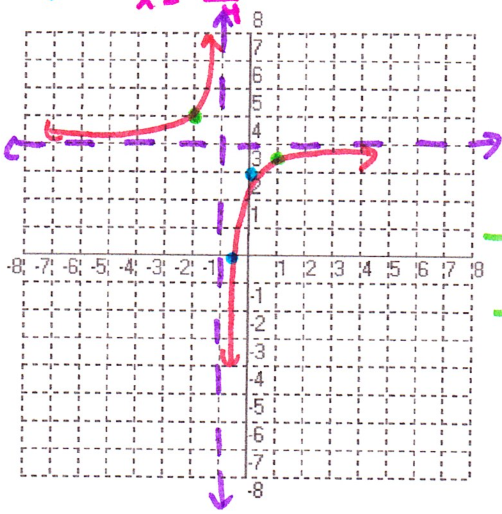
$x=\frac{-3}{4}$

$(-\frac{3}{4},0)$

y-int : $y=\frac{4(0)+3}{0+1}$

$y=3$

$(0,3)$



x	y
1	3.5
-2	$\frac{-5}{-1}=5$

b. $g(x)=\frac{x^2-2x}{x-1}$ $\quad \frac{x(x-2)}{x-1}$

VA : $x=1$

HA : none

SA : $y=x-1$

$$\begin{array}{r|rr|r} 1 & 1 & -2 & 0 \\ & & 1 & \\ \hline & 1 & -1 & \end{array}$$

x-int : $0=\frac{x(x-2)}{x-1}$

$0=x(x-2)$

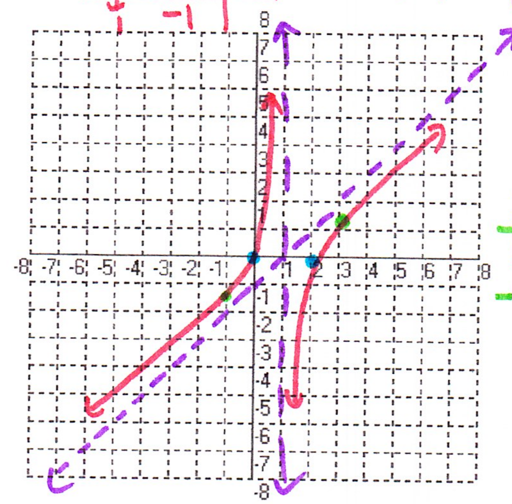
$x=0,2$

$(0,0)$ & $(2,0)$

y-int : $y=\frac{0^2-2(0)}{0-1}$

$y=0$

$(0,0)$



x	y
3	$\frac{3}{2}$
-1	$\frac{3}{-2}$

c. $f(x)=\frac{x+3}{x^2+4x-5}$ $\quad \frac{x+3}{(x+5)(x-1)}$

VA: $x=-5$, $x=1$

HA : $y=0$

x-int : $0=\frac{x+3}{(x+5)(x-1)}$

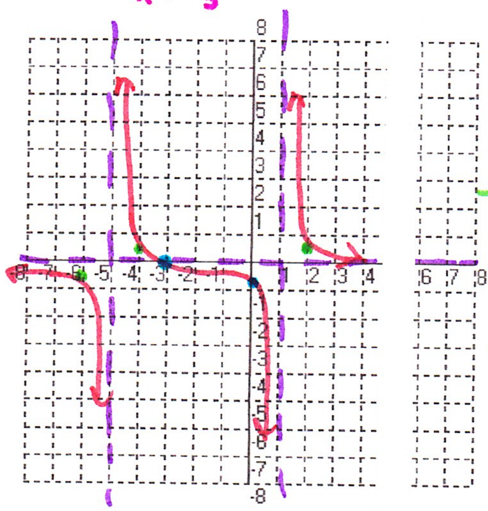
$0=x+3$

$x=-3$

$(-3,0)$

y-int : $y=\frac{0+3}{0+0-5}$ $\quad y=\frac{-3}{5}$

$(0,-\frac{3}{5})$



x	y
2	$\frac{5}{4+8-5}=\frac{5}{7}$
-4	$\frac{-1}{16-16-5}=\frac{1}{5}$
-6	$\frac{-3}{36-24-5}=\frac{-3}{7}$

d. $f(x)=\frac{3x^2+2x-5}{x^2-4}$ $\quad \frac{(3x+5)(x-1)}{(x+2)(x-2)}$

VA: $x=-2$, $x=2$

HA : $y=3$

x-int : $0=\frac{(3x+5)(x-1)}{(x+2)(x-2)}$

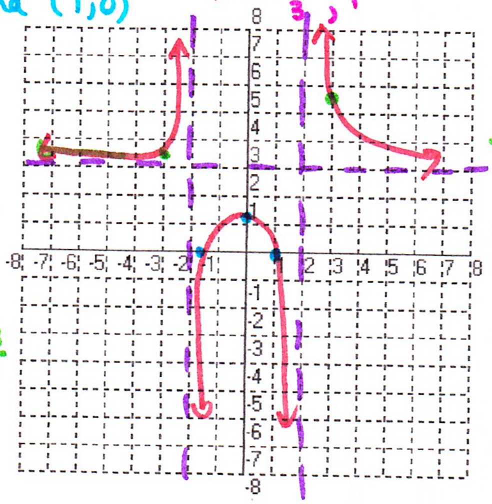
$0=(3x+5)(x-1)$

$x=-\frac{5}{3}, 1$

$(-\frac{5}{3},0)$ and $(1,0)$

y-int : $y=\frac{0+0-5}{0-4}=\frac{5}{4}$

$(0,\frac{5}{4})$



x	y
3	$\frac{27+6-5}{5}=\frac{28}{5}=5\frac{3}{5}$
-3	$\frac{27-6-5}{9-4}=\frac{16}{5}=3\frac{1}{5}$