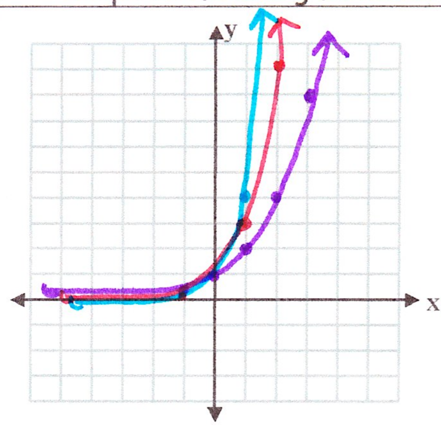
Pre-Calculus Notes
Section 3.1 – Exponential Functions

Name: Key

CONCEPT ONE: Graphing exponential functions.

Example 1: Graph the following functions on the grid provided. Then answer the questions.

a. $y=2^x$
b. $y=3^x$
c. $y=4^x$



A. What happens to the graph as the base increases?
increases @ faster rate

B. What is the y-intercept for each function? WHY?
$(0,1)$ $a^0=1$

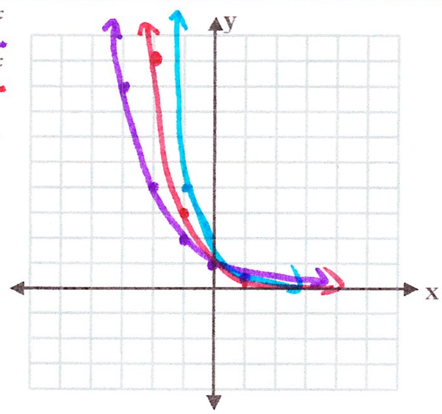
C. What are the x-intercepts? WHY?
none b/c $0 \neq a^n$ ever

D. What is the domain of each function? What is the range?
D: $\mathbb{R}$ R: $(0,\infty)$ OR $y>0$

E. Are the functions increasing or decreasing? One-to-one?
increasing and one-to-one

Example 2: Graph the following functions on the grid provided. Then answer the questions.

a. $y=2^{-x}$
b. $y=3^{-x}$
c. $y=\frac{1}{4^x}$



A. What happens to the graph as the base increases?
decreases @ faster rate

B. What is the y-intercept for each function? WHY?
$(0,1)$

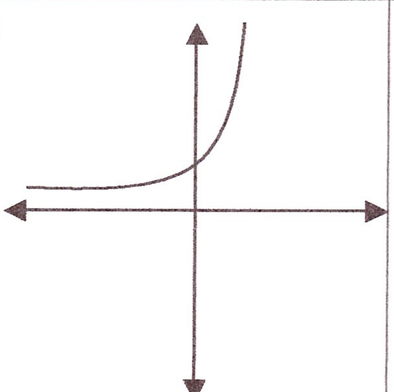
C. What are the x-intercepts? WHY?
none

D. What is the domain of each function? What is the range?
D: $\mathbb{R}$ R: $(0,\infty)$ OR $y>0$

E. Are the functions increasing or decreasing? One-to-one?
decreasing and one-to-one

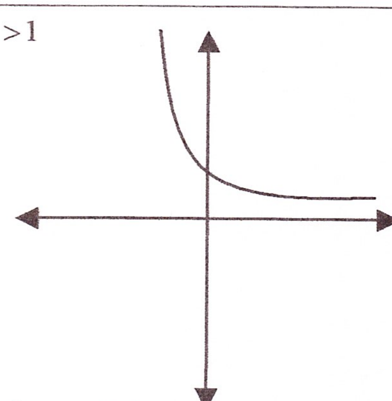
GENERALIZATIONS FOR EXPONENTIAL FUNCTIONS:

Graph of $y=a^x$, $a>1$



Domain: $(-\infty,\infty)$
Range: $(0,\infty)$
Intercept: $(0,1)$
Increasing
x - axis is a horizontal asymptotes
$(a^x \to 0$ as $x \to \infty)$
Continuous

Graph of $y=a^{-x}$, $a>1$



Domain: $(-\infty,\infty)$
Range: $(0,\infty)$
Intercept: $(0,1)$
Decreasing
x - axis is a horizontal asymptotes
$(a^{-x} \to 0$ as $x \to \infty)$
Continuous

CONCEPT TWO: Transformations of Exponential Functions.

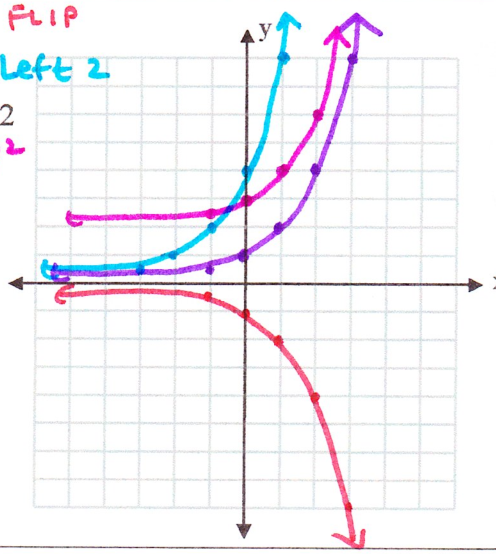
Example 3: Graph the following functions on the grid provided.

a. $y=-2^x$ FLIP

b. $y=2^{x+2}$ Left 2

c. $y=2^x+2$ up 2

$y=2^x$

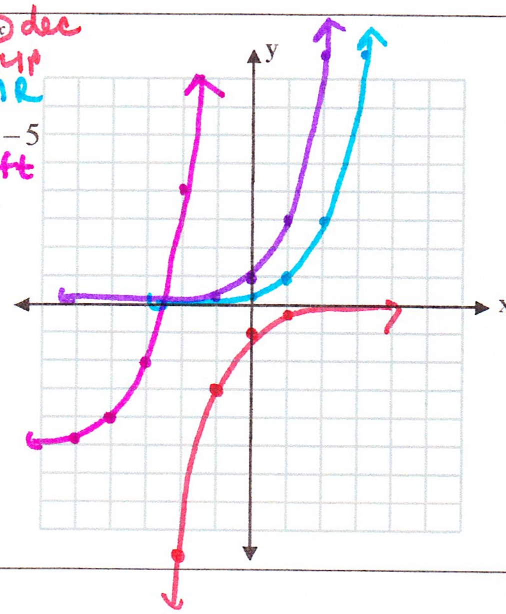


d. $y=-3^{-x}$ dec Flip

e. $y=3^{x-1}$ 1R

f. $y=3^{x+4}-5$ 4 left ↓ 5

$y=3^x$



GENERALIZATIONS FOR TRANSFORMATIONS

Horizontal Shift $y=a^x$ to $y=a^{x+c}$	Vertical Shift $y=a^x$ to $y=a^x+c$
Reflection in x-axis $y=a^x$ to $y=-a^x$	Reflection in y-axis $y=a^x$ to $y=a^{-x}$

CONCEPT THREE: Using the One-to-One Property to solve equations.

Example 4: Solve for x.

a. $9=3^{x+1}$

$3^{2}=3^{x+1}$

$2=x+1$

$\boxed{x=1}$

b. $8^{x-3}=4^{x+1}$

$(2^3)^{x-3}=(2^2)^{x+1}$

$2^{3x-9}=2^{2x+2}$

$3x-9=2x+2$

$\boxed{x=11}$

c. $5^{x+3}=\sqrt{125}$

$5^{x+3}=\sqrt{5^3}$

$5^{x+3}=5^{\frac{3}{2}}$

$x+3=\frac{3}{2}$

$\boxed{x=-\frac{3}{2}}$

d. $\left(\frac{1}{2}\right)^{x^2}=8^{3x+6}$

$(2^{-1})^{x^2}=(2^3)^{3x+6}$

$2^{-x^2}=2^{9x+18}$

$-x^2=9x+18$

$0=x^2+9x+18$

$0=(x+6)(x+3)$

$\boxed{x=-6,-3}$

Examples taken from p.227 #66, 67, 69

1. The population P (in millions) of Russia from 1996 to 2004 can be approximated by the model $P = 152.26e^{-0.0039t}$, where t represents the year, with $t = 6$ corresponding to 1996.

(a) According to the model, is the population of Russia increasing or decreasing? EXPLAIN.

In the form $y = b \cdot a^{-x}$ w/ $a > 1$, therefore, it is decreasing.

(b) Find the population of Russia in 1998 and in 2000.

$1998 \longrightarrow t = 8$ ≈ 148 million

$2000 \longrightarrow t = 10$ ≈ 146 million

(c) Use the model to predict the population of Russia in 2010.

$2010 \longrightarrow t = 20$ ≈ 141 million

2. Let Q represent a mass of a radioactive radium (^{226}Ra) (in grams), whose half-life is 1599 years. The quantity of radium present after t years is $Q = 25\left(\frac{1}{2}\right)^{t/1599}$.

(a) Determine the initial quantity (when $t = 0$).

25 grams

(b) Determine the quantity present after 1000 years.

≈ 16.2 grams

(c) Use a graphing utility to graph the function over the interval $t = 0$ to $t = 5000$.

3. To estimate the amount of defoliation caused by the gypsy moth during a given year, a forester counts the number x of egg masses on $\frac{1}{40}$ of an acre (circle of radius 18.6 feet) in the fall. The percent of defoliation y the next spring is shown in the table.

Egg masses, x	**Percent of defoliation, y**
0	12
25	44
50	81
75	96
100	99

A model for the data is given by $y = \dfrac{100}{1 + 7e^{-0.069x}}$.

(a) Use a graphing utility to create a scatter plot of the data and graph the model in the same window.

(b) Estimate the percent of defoliation if 36 egg masses are counted on $\frac{1}{40}$ acre.

$x = 36$

$\boxed{\approx 63\%}$

(c) You observe that $\frac{2}{3}$ of a forest is defoliated the following spring. Use the graph in part (a) to estimate the number of egg masses per $\frac{1}{40}$ acre.

$\boxed{\approx 38 \text{ egg masses}}$

$y = \frac{2}{3}(100)$ ← put in for y2 and calculate intersection