Pre-Calculus Notes Name: key

Section ~~9.3~~ 3.2 – Logarithmic Functions

Since the exponential function $f(x) = b^x$ is one-to-one, it has an inverse function. The inverse function of an exponential function is called a logarithmic function.

MEMORIZE: If $x = b^y$, then $y = \log_b x$. **AND** If $y = \log_b x$, then $x = b^y$.

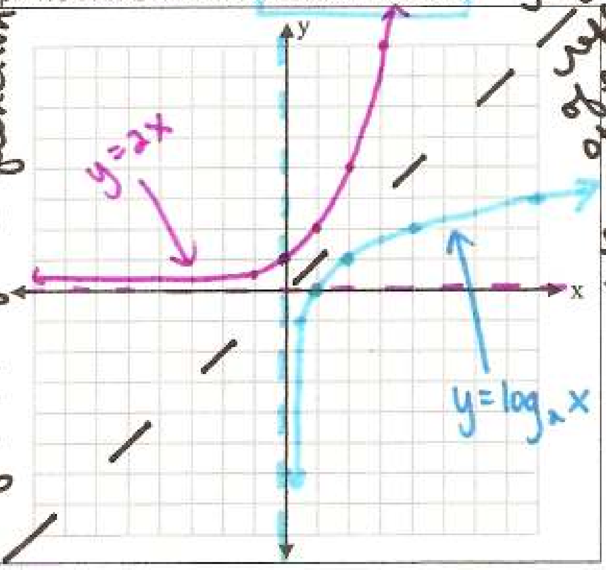
• inverses are reflections of each other across line y=x

EXAMPLE 1: Graph the function in ONE COLOR. Then graph its INVERSE in a SECOND COLOR.

ORIGINAL FUNCTION: $y = 2^x$
Domain: $(-\infty, \infty)$ Range: $(0, \infty)$
X-Intercepts: none
Y-Intercepts: (0, 1)
Increasing or Decreasing? increasing
Equation of Asymptote: $y = 0$

INVERSE FUNCTION: $y = \log_2 x$
Domain: $(0, \infty)$ Range: $(-\infty, \infty)$
X-Intercepts: (1, 0)
Y-Intercepts: none
Increasing or Decreasing? increasing
Equation of Asymptote: $x = 0$

remember → inverse switches the x & y of the original function



MEMORIZE: a logarithm is just an exponent! ☺

A logarithm with a base of 10 is a common logarithm. So, instead of writing $\log_{10} x$, we will write $\log x$.

A logarithm with a base of "e" is a natural logarithm. So, instead of writing $\log_e x$, we will write $\ln x$.

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e \text{ and } e \approx 2.718281828\ldots$$

$$a^x = m$$

exponent
result
base

so... $\log_a m = x$

result
exponent
base

Example 2: Rewrite each expression in logarithmic form.

a. $4^3 = 64$ $\boxed{\log_4 64 = 3}$	b. $10^3 = 1000$ $\log_{10} 1000 = 3$ $\boxed{\log 1000 = 3}$	c. $e^{-2} \approx 0.14$ $\log_e .14 \approx -2$ $\boxed{\ln .14 \approx -2}$

Example 3: Rewrite each expression in exponential form.

a. $\ln 2 \approx 0.70$ $\log_e 2 \approx .70$ $\boxed{e^{.70} \approx 2}$	b. $\log_5 125 = 3$ $\boxed{5^3 = 125}$	c. $\log 0.1 = -1$ $\log_{10} 0.1 = -1$ $\boxed{10^{-1} = .1}$

Example 4: Use the definition of logarithmic function to evaluate each logarithm. NO CALCULATOR!

a. $\log_2 32$ $2^5 = 32$ $\boxed{5}$	b. $\log_3 1$ $3^0 = 1$ $\boxed{1}$	c. $\log_4 2$ $4^{\frac{1}{2}} = 2$ $\boxed{\frac{1}{2}}$	d. $\log_{10} \frac{1}{100}$ $10^{-2} = \frac{1}{100}$ $\boxed{-2}$

Example 5: Evaluate with the calculator. Round to 3 decimal places.

a. $\log 25$ $\boxed{1.398}$	b. $\ln 0.34$ $\boxed{-1.079}$	c. $\log x = 2.014$ DO 2nd log 2.014 $\boxed{103.276}$	d. $\ln x = -4$ DO 2nd ln -4 $\boxed{.018}$	e. $\log x = 0$ DO 2nd log 0 $\boxed{1.000}$

MEMORIZE: Change of Base Formula

The Change of Base Formula is used in order to evaluate a logarithm with a base other than 10 in the calculator. The Change-of-Base Formula is $\log_b x = \frac{\log_a x}{\log_a b}$

Example 6: Use the change of base formula to evaluate to 3 decimal places.

a. $\log_2 15$ $\frac{\log 15}{\log 2}$ $\boxed{3.907}$	b. $\log_{\frac{1}{4}} 20$ $\frac{\log 20}{\log \frac{1}{4}}$ $\boxed{-2.161}$	c. $\log_{\sqrt{6}} 1.5$ $\frac{\log 1.5}{\log \sqrt{6}}$ $\boxed{.453}$