

Section 3.3 - Properties of Logarithms

MEMORIZE: Properties of Logarithms

1. Product Property: $\log_a(uv) = \log_a u + \log_a v$

ex. $\log_4(3x) = \log_4 3 + \log_4 x$

2. Quotient Property: $\log_a \frac{u}{v} = \log_a u - \log_a v$

ex. $\log_5 \frac{10}{y} = \log_5 10 - \log_5 y$

3. Power Property: $\log_a u^n = n \log_a u$

ex. $\log_7 x^{12} = 12 \log_7 x$

We will later use these properties to solve equations involving logarithms.

Example 1: Use the properties of logarithms to EXPAND each of the following expressions.

<p>a. $\log_b \frac{3x}{z}$</p> <p>$\log_b 3x - \log_b z$</p> <p>$\log_b 3 + \log_b x - \log_b z$</p>	<p>b. $\log_b \sqrt[5]{x^2 y}$</p> <p>$\log_b (x^2 y)^{\frac{1}{5}}$</p> <p>$\log_b x^{\frac{2}{5}} y^{\frac{1}{5}}$</p> <p>$\log_b x^{\frac{2}{5}} + \log_b y^{\frac{1}{5}}$</p> <p>$\frac{2}{5} \log_b x + \frac{1}{5} \log_b y$</p>	<p>c. $\ln 4xy\sqrt{z}$</p> <p>$\ln 4 + \ln x + \ln y + \ln \sqrt{z}$</p> <p>$\ln 4 + \ln x + \ln y + \ln z^{\frac{1}{2}}$</p> <p>$\ln 4 + \ln x + \ln y + \frac{1}{2} \ln z$</p>
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Example 2: Use the properties of logarithms to CONDENSE each of the following expressions.

<p>a. $3 \log x + \log 3 - 4 \log y$</p> <p>$\log x^3 + \log 3 - \log y^4$</p> <p>$\log x^{3 \cdot 3} - \log y^4$</p> <p>$\log \frac{3x^3}{y^4}$</p>	<p>b. $2 \log_5 x - 3 \log_5 2 + 2 \log_5 4$</p> <p>$\log_5 x^2 - \log_5 2^3 + \log_5 4^2$</p> <p>$\log_5 x^2 - \log_5 8 + \log_5 16$</p> <p>$\log_5 \frac{x^2}{8} + \log_5 16$</p> <p>$\log_5 \frac{16x^2}{8}$</p> <p>$\log_5 2x^2$</p>	<p>c. $\frac{1}{2} \ln 9 + 3 \ln(2x) + 2 \ln(y^2)$</p> <p>$\ln 9^{\frac{1}{2}} + \ln(2x)^3 + \ln(y^2)^2$</p> <p>$\ln 3 + \ln 8x^3 + \ln y^4$</p> <p>$\ln 3 \cdot 8x^3 \cdot y^4$</p> <p>$\ln 24x^3 y^4$</p>
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Example 3: Use the properties of logarithms to evaluate each expression WITHOUT a calculator.

<p>a. $\log_5 \sqrt[3]{5}$</p> <p>$\log_5 5^{\frac{1}{3}}$</p> <p>$\frac{1}{3} \log_5 5 = 1$</p> <p>$\frac{1}{3} \cdot 1$</p> <p>$\frac{1}{3}$</p>	<p>b. $\ln e^6 - \ln e^2$</p> <p>$\ln \frac{e^6}{e^2}$</p> <p>$\ln e^4$</p> <p>$4 \ln e = 1$</p> <p>$4 \cdot 1$</p> <p>4</p>	<p>c. $\log_4 8$</p> <p>$\log_4 2^3$</p> <p>$3 \log_4 2 = \frac{1}{2}$</p> <p>$3 \cdot \frac{1}{2}$</p> <p>$\frac{3}{2}$</p>
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