Pre-Calculus Notes

Name: key

Section 3.4 – Solving Equations with Logarithms

Strategies for Solving Exponential and Logarithmic Equations:

1. Rewrite the original equation in a form that allows the use of *One-to-One Properties* of exponential or logarithmic functions.
2. Rewrite an exponential equation in logarithmic form and solve.
3. Rewrite a logarithmic equation in exponential form and solve. You may need to first use the properties of logarithms to condense the logarithmic expression.

Example 1: Solve each equation. **HINT: Rewrite in logarithmic form.**

a. $3^x + 5 = 10$

$3^x = 5$

$\log_3 5 = x$

$x = \frac{\log 5}{\log 3}$

$\boxed{x \approx 1.465}$

b. $4^{2x-1} - 6 = 16$

$4^{2x-1} = 22$

$\log_4 22 = 2x - 1$

$x = \frac{\log_4 22 + 1}{2}$

$x = \frac{\frac{\log 22}{\log 4} + 1}{2}$

$\boxed{x \approx 1.615}$

Example 2: Solve each equation. **HINT: Rewrite in exponential form.**

a. $\log_x 125 = 3$

$\sqrt[3]{x^3} = \sqrt[3]{125}$

$\boxed{x = 5}$

b. $\log_x\left(\frac{9}{4}\right) = \frac{1}{2}$

$\left(x^{\frac{1}{2}}\right)^2 = \left(\frac{9}{4}\right)^2$

$\boxed{x = \frac{81}{16}}$

Example 3: Round to 3 decimal places when needed.

a. $5 + 2\ln x = 4$

$2\ln x = -1$

$\ln x = -\frac{1}{2}$

$e^{-\frac{1}{2}} = x$

$\boxed{x \approx .607}$

b. $e^x + 5 = 60$

$e^x = 55$

$\ln 55 = x$

$\boxed{x \approx 4.007}$

c. $4^{2x-1} = 5^{x+2}$

$\log 4^{2x-1} = \log 5^{x+2}$

$(2x-1)\log 4 = (x+2)\log 5$

$2x\log 4 - \log 4 = x\log 5 + 2\log 5$

$2x\log 4 - x\log 5 = 2\log 5 + \log 4$

$x(2\log 4 - \log 5) = 2\log 5 + \log 4$

$x = \frac{2\log 5 + \log 4}{2\log 4 - \log 5}$

$\boxed{x \approx 3.959}$

d. $\log_3(5x-1) = \log_3(x+7)$

$5x - 1 = x + 7$

$4x = 8$

$\boxed{x = 2}$

e. $2\log_5(3x) = 4$

$\log_5(3x) = 2$

$5^2 = 3x$

$25 = 3x$

$\boxed{x \approx 8.333}$

f. $\log(5x) + \log(x-1) = 2$

$\log 5x(x-1) = 2$

$10^2 = 5x(x-1)$

$100 = 5x^2 - 5x$

$0 = 5x^2 - 5x - 100$

$0 = x^2 - x - 20$

$0 = (x+4)(x-5)$

$\cancel{x = -4}$, $\boxed{x = 5}$

$\boxed{x = 5}$