Pre-Calculus Notes

Name: Key

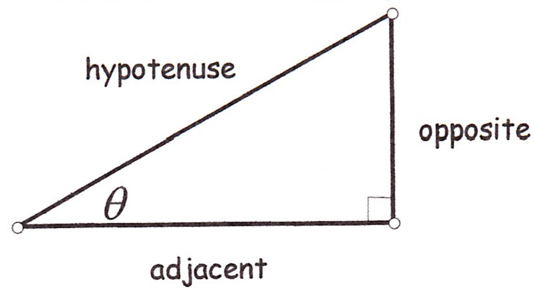
Section 4.1 - The 6 Trigonometric Functions

PART ONE: Definitions Using Right Triangles: **SOH-CAH-TOA**

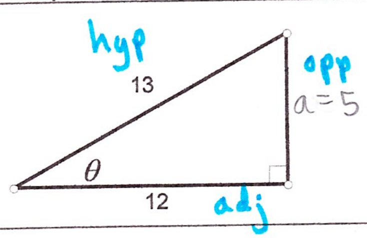
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\qquad \csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\qquad \sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $\qquad \cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$



Example 1: Find the values of the six trigonometric functions of θ.



$a^2 + 12^2 = 13^2$

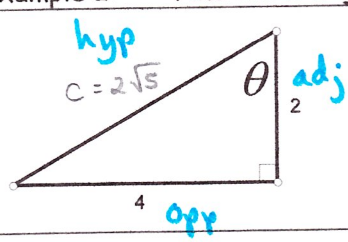
$a^2 = 25$

$a = 5$

$\sin\theta = \frac{5}{13}$ $\quad \cos\theta = \frac{12}{13}$ $\quad \tan\theta = \frac{5}{12}$

$\csc\theta = \frac{13}{5}$ $\quad \sec\theta = \frac{13}{12}$ $\quad \cot\theta = \frac{12}{5}$

Example 2: Find the 6 trig. values of the angle shown. Give exact values, simplified.



$2^2 + 4^2 = c^2$

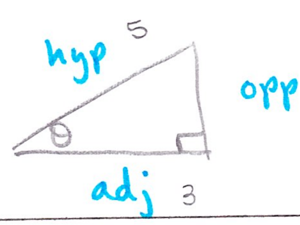
$c^2 = 20$

$c = 2\sqrt{5}$

$\sin\theta = \frac{2\sqrt{5}}{5}$ $\quad \frac{4}{2\sqrt5}\cdot\frac{\sqrt5}{\sqrt5}$ $\quad \cos\theta = \frac{\sqrt{5}}{5}$ $\quad \frac{2}{2\sqrt5} = \frac{1}{\sqrt5}$ $\quad \tan\theta = 2$

$\csc\theta = \frac{\sqrt{5}}{2}$ $\quad \sec\theta = \sqrt{5}$ $\quad \cot\theta = \frac{1}{2}$

Example 3: Sketch a right triangle corresponding to the given trig. function and its value. Find the third side and the other trig. values.



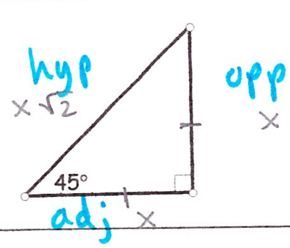
opp 4 (triple)

$\sin\theta = \frac{4}{5}$ $\quad \cos\theta = \frac{3}{5}$ $\quad \tan\theta = \frac{4}{3}$

$\csc\theta = \frac{5}{4}$ $\quad \sec\theta = \frac{5}{3}$ $\quad \cot\theta = \frac{3}{4}$

You will need to have the following trig value <u>MEMORIZED</u> very "SOON".

Example 4: Find the values of the six trigonometric functions of θ in a $45^\circ - 45^\circ - 90^\circ$ triangle.



opp x

$x^2 + x^2 = c^2$

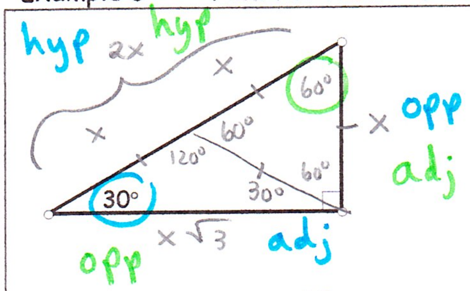
$2x^2 = c^2$

$c = x\sqrt{2}$

$\sin 45^\circ = \frac{\sqrt{2}}{2}$ $\quad \frac{x}{x\sqrt2} = \frac{1}{\sqrt2}$ $\quad \cos 45^\circ = \frac{\sqrt{2}}{2}$ $\quad \tan 45^\circ = 1$ $\quad \frac{x}{x}$

$\csc 45^\circ = \sqrt{2}$ $\quad \sec 45^\circ = \sqrt{2}$ $\quad \cot 45^\circ = 1$

Example 5: Find the values of the six trigonometric functions of θ in a $30^\circ - 60^\circ - 90^\circ$ triangle.



$x^2 + a^2 = (2x)^2$

$a^2 = 3x^2 \qquad a = x\sqrt{3}$

$\sin 30^\circ = \underline{\frac{1}{2}}$ $\quad \frac{x}{2x}$ $\qquad \cos 30^\circ = \underline{\frac{\sqrt{3}}{2}}$ $\quad \frac{x\sqrt{3}}{2x}$ $\qquad \tan 30^\circ = \underline{\frac{\sqrt{3}}{3}}$ $\quad \frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\csc 30^\circ = \underline{2} \qquad \sec 30^\circ = \underline{\frac{2\sqrt{3}}{3}} \qquad \cot 30^\circ = \underline{\sqrt{3}}$

$\sin 60^\circ = \underline{\frac{\sqrt{3}}{2}} \qquad \cos 60^\circ = \underline{\frac{1}{2}} \qquad \tan 60^\circ = \underline{\sqrt{3}}$

$\csc 60^\circ = \underline{\frac{2\sqrt{3}}{3}} \qquad \sec 60^\circ = \underline{2} \qquad \cot 60^\circ = \underline{\frac{\sqrt{3}}{3}}$

Could we summarize the above information into a single chart? Yes!!!!!!!!!!!!!!! * teach "hand jive" here

	$\frac{\pi}{6}$ or 30°	$\frac{\pi}{4}$ or 45°	$\frac{\pi}{3}$ or 60°
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\tan\theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Example 6: Fill in the blanks with the angle measure or function value.

Function	θ in degrees	θ in radians	Value
sec so $\cos\theta = \frac{\sqrt{2}}{2}$	45°	$\frac{\pi}{4}$	$\sqrt{2}$
tan	45°	$\frac{\pi}{4}$	1
cos	60°	$\frac{\pi}{3}$	$\frac{1}{2}$
csc so $\sin\theta = \frac{1}{2}$	30°	$\frac{\pi}{6}$	2

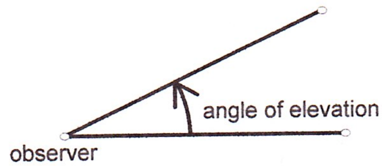
Now let's do these on the CALCULATOR...

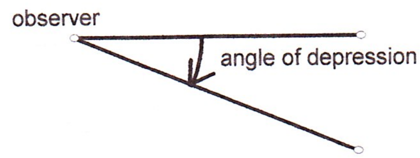
Example 2: Calculate to 4 decimal places.

a. $\sin 25^\circ$ 0.4226	b. $\cos 32.4^\circ$ 0.8443	c. $\tan 40^\circ 15' 10''$ 0.8466
d. $\csc 85^\circ = \frac{1}{\sin 85^\circ}$ 1.0038	e. $\sec 75^\circ 30' = \frac{1}{\cos 75^\circ 30'}$ 3.9939	f. $\cot 56.27^\circ$ $\frac{1}{\tan 56.27^\circ}$ 0.6677

Applications – Solving Right Triangles

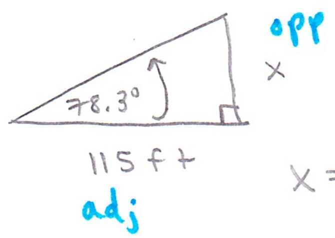
Do you remember angle of elevation and angle of depression?





Example 3:

A surveyor is standing 115 feet from the base of the Washington Monument. The surveyor measures the angle of elevation to the top of the monument as 78.3°. How tall is the Washington Monument?

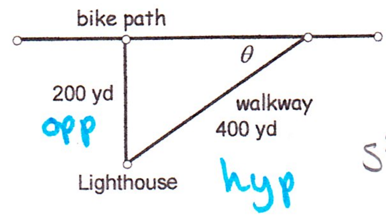


$$\tan 78.3^\circ = \frac{x}{115}$$

$$x = 115 \tan 78.3^\circ$$

$$\boxed{\approx 555.3 \text{ ft}}$$

An historic lighthouse is 200 yards from a bike path along the lake. A walkway to the lighthouse is 400 yards long. Find the acute angle between the bike path and the walkway.

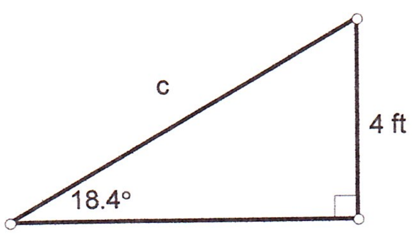


$$\sin \theta = \frac{200}{400}$$

$$\sin \theta = \frac{1}{2}$$

so $\boxed{\theta = 30^\circ}$

Example 5: Solve for c.



$$\frac{\sin 18.4^\circ}{1} = \frac{4}{c}$$

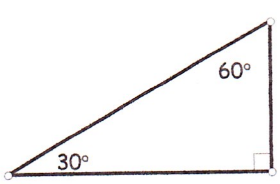
$$c \sin 18.4^\circ = 4$$

$$c = \frac{4}{\sin 18.4^\circ}$$

$$\boxed{c \approx 12.67 \text{ ft}}$$

PART TWO: Let's talk now about the topic of "CO-FUNCTIONS".

Think about the $30^\circ - 60^\circ - 90^\circ$ triangle.



$\sin 30^\circ = \underline{\frac{1}{2}}$ ⟷ $\cos 60^\circ = \underline{\frac{1}{2}}$

$\tan 30^\circ = \underline{\frac{\sqrt{3}}{3}}$ ⟷ $\cot 60^\circ = \underline{\frac{\sqrt{3}}{3}}$

$\sec 30^\circ = \underline{\frac{2\sqrt{3}}{3}}$ ⟷ $\csc 60^\circ = \underline{\frac{2\sqrt{3}}{3}}$

WHY are those values equal? angles are complementary and b/c for each, the opp/adj switch

WHEN will those function values be equal? When you have co-functions!

In general, it can be shown from the right triangle definitions that **co-functions of complementary angles are equal.** That is, if θ is an acute angle, the following relationships are true:

$$\sin\theta = \cos(90^\circ - \theta) \ \ or \ \ \cos\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\theta = \cot(90^\circ - \theta) \ \ or \ \ \cot\left(\frac{\pi}{2} - \theta\right)$$

$$\sec\theta = \csc(90^\circ - \theta) \ \ or \ \ \csc\left(\frac{\pi}{2} - \theta\right)$$

So, let's try this again. What functions are CO-FUNCTIONS?

sine **AND** Cosine tangent **AND** Cotangent secant **AND** Cosecant

WHEN will the co-function values be equal? when angles are complementary

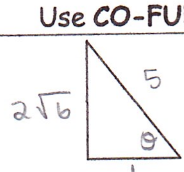
We also need to start looking at some of the TRIG. IDENTITIES.

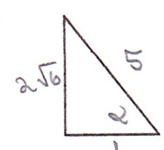
Look at the following and discuss WHY they are true.

Fundamental Identities:	
Reciprocal Identities:	**Quotient Identities:**
$\sin\theta = \dfrac{1}{\csc\theta}$; $\csc\theta = \dfrac{1}{\sin\theta}$	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
$\cos\theta = \dfrac{1}{\sec\theta}$; $\sec\theta = \dfrac{1}{\cos\theta}$	$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$
$\tan\theta = \dfrac{1}{\cot\theta}$; $\cot\theta = \dfrac{1}{\tan\theta}$	

Example 1: Use **CO-FUNCTIONS** and **TRIG IDENTITIES** to find the other values.

If $\sec\theta = 5$ and $\tan\alpha = 2\sqrt{6}$, then find the following...





a. $\cos\theta = \dfrac{1}{\sec\theta} = \boxed{\dfrac{1}{5}}$

b. $\cot\alpha = \dfrac{1}{\tan\alpha} = \dfrac{1}{2\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} = \boxed{\dfrac{\sqrt{6}}{12}}$

c. $\csc(90^\circ - \theta) = \sec\theta = \boxed{5}$

d. $\cot(90^\circ - \alpha) = \tan\alpha = \boxed{2\sqrt{6}}$

e. $\csc\theta = \dfrac{1}{\sin\theta}$

f. $\tan(90^\circ - \alpha) = \cot\alpha = \boxed{\dfrac{\sqrt{6}}{12}}$