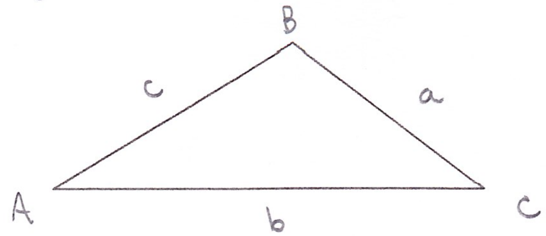
Pre-Calculus Notes   Name: Key

Section 6.1 – Intro to the Law of Sines & Section 6.2 – Areas of Triangles

The LAW of SINES… what is it's use? solving for sides and angles of Δ when given 2 angles and 1 side OR 2 sides and non-included $\angle$



AAS
ASA
SSA

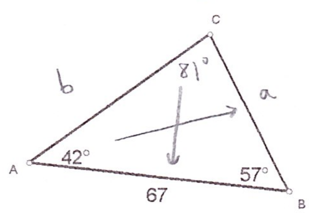
MEMORIZE: THE LAW OF SINES

* to be used given AAS, ASA, or SSA

For ANY triangle ABC, where a, b, and c are the lengths of the sides OPPOSITE the angles with measures A, B, and C (respectively)… $\boxed{\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}}$. ← Law of Sines

Example 1: Solve ΔABC.

a. Note: We are given **ASA** here.



$C = 180 - 42 - 57$

$\boxed{C = 81°}$

$\dfrac{\sin 81°}{67} = \dfrac{\sin 42°}{a}$

$a \sin 81° = 67 \sin 42°$

$a = \dfrac{67 \sin 42°}{\sin 81°}$

$\boxed{a \approx 45}$

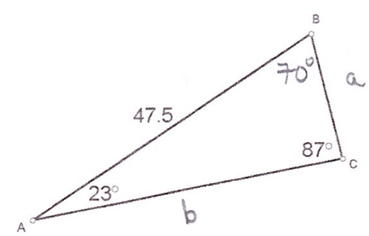
$\dfrac{\sin 57°}{b} = \dfrac{\sin 81°}{67}$

$b \sin 81° = 67 \sin 57°$

$b = \dfrac{67 \sin 57°}{\sin 81°}$

$\boxed{b \approx 57}$

b. Note: We are given **AAS** here.



$\boxed{B = 70°}$

$\dfrac{\sin 23°}{a} = \dfrac{\sin 87°}{47.5}$

$a \sin 87° = 47.5 \sin 23°$

$a = \dfrac{47.5 \sin 23°}{\sin 87°}$

$\boxed{a \approx 18.6}$

$\dfrac{\sin 70°}{b} = \dfrac{\sin 87°}{47.5}$

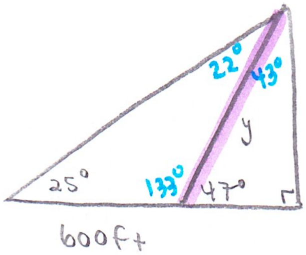
$b \sin 87° = 47.5 \sin 70°$

$b = \dfrac{47.5 \sin 70°}{\sin 87°}$

$\boxed{b \approx 44.7}$

Example 2: WORD PROBLEM.

A ship is moving in a straight line towards the Point Cove lighthouse. The measure of the angle of elevation from the bridge of the ship to the lighthouse beacon is 25°. Later, from a point 600 feet closer, the angle of elevation is 47°. To the nearest foot, how high is the beacon above the level of the bridge of the ship?



$$\frac{\sin 25^\circ}{y} = \frac{\sin 22^\circ}{600}$$

$$y = \frac{600 \sin 25^\circ}{\sin 22^\circ}$$

Store in calculator!

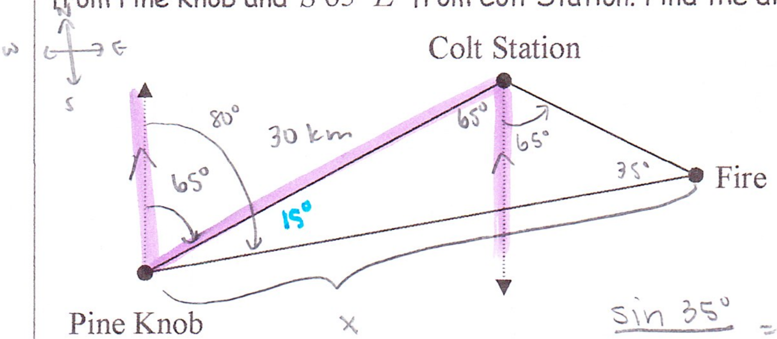
$$\sin 47^\circ = \frac{x}{y}$$

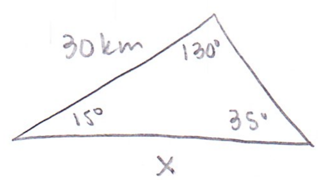
$$x = y \sin 47^\circ$$

$$x \approx \boxed{495 \text{ ft}}$$

Example 3: WORD PROBLEM.

The bearing from the Pine Knob fire tower to the Colt Station fire tower is $N \ 65^\circ \ E$, and the two towers are 30 kilometers apart. A fire spotted by rangers in each tower has a bearing of $N\,80^\circ\,E$ from Pine Knob and $S\,65^\circ\,E$ from Colt Station. Find the distance of the fire from the Pine Knob tower.





$$\frac{\sin 35^\circ}{30} = \frac{\sin 130^\circ}{x}$$

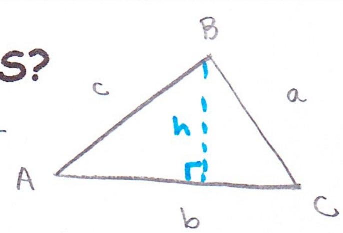
$$x = \frac{30 \sin 130^\circ}{\sin 35^\circ}$$

$$x \approx \boxed{40 \text{ km}}$$

Do you remember the formula for finding the area of a triangle given **SAS?**

$A = \frac{1}{2}bh$ ⟵ $\sin A = \frac{h}{c}$ ⟵

$K = \frac{1}{2} b \cdot c \sin A$ $\quad$ $h = c \sin A$



The area, K of triangle ABC is given by any one of these formulas:

$$K = \frac{1}{2}bc\sin A \qquad K = \frac{1}{2}ac\sin B \qquad K = \frac{1}{2}ab\sin C$$

We also have a formula for finding the area of a triangle given **SSS.** (The Greek mathematician Heron developed the formula - hence it is called **HERONS' AREA FORMULA**.)

The area, K of triangle ABC is given by:

$K=\sqrt{s(s-a)(s-b)(s-c)}$, where $s=\dfrac{a+b+c}{2}$. s is called the semi-perimeter of the triangle.

Example 1: Determine the area of ΔDEF to the nearest square inch. DRAW A PICTURE.

$d=15.2$, $e=22.7$, and $f=8.9$

$S=\dfrac{15.2+22.7+8.9}{2}$

$S=23.4$

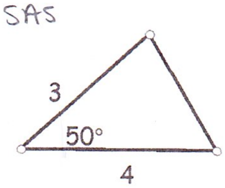
store! (saves time)
(as x)

$K=\sqrt{x(x-15.2)(x-22.7)(x-8.9)}$

$\boxed{K \approx 44 \text{ in}^2}$

Example 2: Which formula would you use to find the area of the following triangles? Find the area.

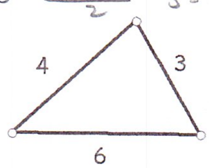
a. SAS



$K=\frac{1}{2}\cdot 3\cdot 4\cdot \sin 50°$

$\boxed{K \approx 4.6 \text{ u}^2}$

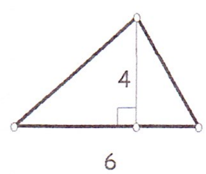
b. $S=\frac{4+3+6}{2}$ $S=6.5$



$K=\sqrt{s(s-4)(s-3)(s-6)}$

$\boxed{K \approx 5.3 \text{ u}^2}$

c.

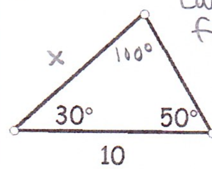


$K=\frac{1}{2}bh$

$K=\frac{1}{2}\cdot 6\cdot 4$

$\boxed{K=12 \text{ u}^2}$

d.



Law of Sines to find side length!

$\dfrac{\sin 50°}{x}=\dfrac{\sin 100°}{10}$

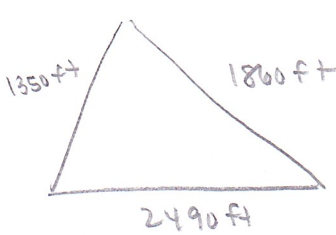
$x=\dfrac{10\sin 50°}{\sin 100°}$ store!

$K=\frac{1}{2}\cdot 10\cdot x\cdot \sin 30°$

$\boxed{K \approx 19.4 \text{ u}^2}$

Example 3:

You want to buy a triangular lot measuring 1350 feet by 1860 feet by 2490 feet. The price of the land is \$2200 per acre. How much does the land cost? (1acre = 43,560 square feet)



$S=\dfrac{P}{2}$

$S=2850$

$K=\sqrt{2850\cdot 1500\cdot 990\cdot 360}$

$K \approx \dfrac{1234345.981 \text{ ft}^2}{43560 \text{ ft}^2}$

$K \approx 28.33668459$ acres $\times$ \$2200

$\boxed{\text{Cost} \approx \$62,340.71}$