Pre-Calculus Notes Name: Key

Section 9.1 – Sequences and Series DAY ONE

Sequence: a listing of numbers, may or may not have a pattern

Example 1) $-3,\ 7,\ 0,\ 2,\ 100$

$a_1\ \ a_2\ \ a_3\ \ a_4\ \ a_5$ ---- Read "a sub 5" – the 5th term of the sequence

a_n: the n^{th} term of sequence

Can you write other terms using the "nth" term?

Example 2) 5, 10, 15, 20, 25,...

a_{n-1} a_n a_{n+1} a_{n+2} a_{n+3}

Can you find a "formula" to find any term of the above sequence?

a_n = $5n$, therefore a_{50} = 250 and a_{100} = 500

For the following, find the first 5 terms and the indicated term.

Example 3) $a_n = 5n-1$

$a_1 = 5(1) - 1 = 4$

$a_2 = 5(2) - 1 = 9$

$a_3 = 5(3) - 1 = 14$

$a_4 = 5(4) - 1 = 19$

$a_5 = 5(5) - 1 = 24$

$a_{10} = 5(10) - 1 = 49$

Example 4) $a_n = \dfrac{(-2)^n}{n^2}$

$a_1 = \dfrac{(-2)^1}{1^2} \Rightarrow \dfrac{-2}{1} \Rightarrow -2$

$a_2 = \dfrac{(-2)^2}{2^2} \Rightarrow \dfrac{4}{4} \Rightarrow 1$

$a_3 = \dfrac{(-2)^3}{3^2} \Rightarrow \dfrac{-8}{9}$

$a_4 = \dfrac{(-2)^4}{4^2} \Rightarrow \dfrac{16}{16} = 1$

$a_5 = \dfrac{(-2)^5}{5^2} \Rightarrow \dfrac{-32}{25}$

$a_{10} = \dfrac{(-2)^{10}}{10^2} \Rightarrow \dfrac{1024}{100} \Rightarrow \dfrac{256}{25}$

Could you use the table feature of your calculator here? How could we get fractional answers for (Hint: put the numerator in y_1 and the denominator in y_2.) cool trick! ☺

We call the above formulas **EXPLICIT FORMUALAS** because we use the ***number of a term*** to find the term.

Not all sequences have explicit formulas. Consider the following sequence. What is its pattern and what is the 7th term?

Example 4) 1, 1, 2, 3, 5, 8, 13, 21 Name? (Fibonacci sequence)

Pattern? the next term of the sequence is the sum of previous 2 terms

$a_{n+1} =$ $a_n + a_{n-1}$

We call the above formula **RECURSIVE** because we use ***previous terms*** to find a particular term.

Use the following recursive formulas to write the first 5 terms of the sequence and an explicit formula.

Example 5) $a_1 = 13,\ a_{n+1} = a_n + 10$

$a_1 =$ 13

$a_2 =$ 13 + 10 ⇒ 23

$a_3 =$ 23 + 10 ⇒ 33

$a_4 =$ 33 + 10 ⇒ 43

$a_5 =$ 43 + 10 ⇒ 53

$a_n =$ 10n + 3 or 10(n−1) + 13

Example 6) $a_1 = 10,\quad a_{k+1} = 10a_k$

$a_1 =$ 10

$a_2 =$ $10a_1 = 10 \cdot 10 \Rightarrow 100$

$a_3 =$ $10a_2 = 10 \cdot 100 \Rightarrow 1{,}000$

$a_4 =$ $10a_3 = 10 \cdot 1000 \Rightarrow 10{,}000$

$a_5 =$ $10a_4 = 10 \cdot 10000 \Rightarrow 100{,}000$

$a_n =$ $10a_{n-1}$ (recursive) or 10^n (explicit)

Many sequences have both explicit and recursive formulas. Some only have a recursive and some only an explicit. Consider the following sequences and give (a) a recursive formula and/or (b) an explicit formula.

Example 7) 1, 11, 21, 31, 41 (+10, +10, +10)

(a) RECURSIVE FORMULA: $a_{n+1} =$ $a_n + 10$, $a_1 = 1$

(b) EXPLICIT FORMUAL: $a_n =$ $10(n-1) + 1$ or $10n - 9$

Note: The pairing here is (1,1), (2,11), (3,21), (4,31) - this is a linear function. Hence the equation must be in the y = mx + b format!

powers of 4

$m = 10$

$y = 10x + b$

$1 = 10(1) + b$

$b = -9$

Example 8) 4, 16, 64, 256, 1024 (·4, ·4, ·4)

(a) RECURSIVE FORMULA: $a_{n+1} =$ $4a_n$, $a_1 = 4$

(b) EXPLICIT FORMUAL: $a_n =$ 4^n

Example 9) $\frac{1}{1}, \frac{-1}{8}, \frac{1}{27}, \frac{-1}{64}, \frac{1}{125}, \frac{-1}{216}$

$1^3 \quad 2^3 \quad 3^3 \quad 4^3 \quad 5^3$

(a) RECURSIVE FORMULA: $a_{n+1} =$ none

(b) EXPLICIT FORMUAL: $a_n =$ $\frac{(-1)^{n-1}}{n^3}$ or $\frac{(-1)^{n+1}}{n^3}$

Example 10) $1\frac{1}{2}, 1\frac{1}{4}, 1\frac{1}{8}, 1\frac{1}{16}, 1\frac{1}{32}, 1\frac{1}{64}$

$2^1 \quad 2^2 \quad 2^3 \quad 2^4 \quad 2^5$

$\frac{3}{2} \quad \frac{5}{4} \quad \frac{9}{8} \quad \frac{17}{16} \quad \frac{33}{32} \quad \frac{65}{64}$

(a) RECURSIVE FORMULA: $a_{n+1} =$ none

(b) EXPLICIT FORMUAL: $a_n =$ $1+\left(\frac{1}{2}\right)^n$ or $1+\frac{1}{2^n}$

Example 11)

A deposit of $100 is make each month in an account that earns 12% interest compounded monthly. The balance in the account after n months is given by

$$A_n = 100(101)\left[(1.01)^n - 1\right]$$

Hint: use your calculator!!!!!

a. Write the first 5 terms. $101.00, $203.01, $306.04, $410.10, $515.20

b. What is the balance after 5 years? $8248.64

5 years = 60 months so I want 60th term $n=60$

c. What is the balance after 20 years? $99,914.79

20 years = 240 months so $n = 240$