Pre-Calculus Notes Name: Key
Section 9.1 – Sequences and Series DAY TWO

Now let's look at the sum of a sequence - called a SERIES.

Consider the following sequence.
(A) What is its explicit formula?
(B) Write the sequence as a series - sum of the 4 terms.
(C) Find the actual sum of the series.

Ex. 1. $\overset{3\cdot 1}{3}, \overset{3\cdot 2}{6}, \overset{3\cdot 3}{9}, \overset{3\cdot 4}{12}$ (+3, +3, +3)

(A) $a_n =$ 3n

(B) Series: 3 + 6 + 9 + 12 (C) S_4 (sum of 4 terms) = 30

Now let's look at the shorthand notation developed to indicate a series.

$$\sum_{n=1}^{4} 3n = 3\cdot 1 + 3\cdot 2 + 3\cdot 3 + 3\cdot 4 \qquad S_4 = 30$$

We call this form SUMMATION NOTATION or SIGMA NOTATION

Here, n is the *index of summation* (the variable here is n, but can be any letter of the alphabet)
1 is the *lower limit of summation* (this is usually *one* but can be any whole number)
4 is the *upper limit of summation* (this is usually a whole number, but can be ∞)

Find the following series in EXPANDED FORM first and then find the actual *sum of the series*.

Ex. 2. $$\sum_{k=1}^{5} 3n-1 = \overset{3\cdot 1-1}{2} + \overset{3\cdot 2-1}{5} + \overset{3\cdot 3-1}{8} + \overset{3\cdot 4-1}{11} + \overset{3\cdot 5-1}{14} \qquad S_5 = 40$$

Ex. 3. $$\sum_{i=3}^{7} 1+i^2 = \overset{1+3^2}{10} + \overset{1+4^2}{17} + \overset{1+5^2}{26} + \overset{1+6^2}{37} + \overset{1+7^2}{50} \qquad S_5 = 140$$

Now, lets look at how we can use the calculator to get the sum of a series

Calculator: sum (seq ($1+x^2$, x, 3, 7, 1) = 140
List,Math List,OPS
2nd STAT
eq. var. limits of summation go by ones

OR

If you have put $1+x^2$ into y_1

$y_1 = 1+x^2$

Calculator: sum (seq (y_1, x, 3, 7, 1) = 140
Vars,Y-Vars

Now, let's reverse the process and go from expanded form back to Sigma Notation

Ex. 4. $\frac{1}{3(1)+5}+\frac{1}{3(2)+5}+\frac{1}{3(3)+5}+\dots+\frac{1}{3(9)+5} = \sum_{i=1}^{9} \frac{1}{3i+5}$

Ex. 5. $\left[3\left(\frac{1}{5}\right)+1\right]+\left[3\left(\frac{2}{5}\right)+2\right]+\dots+\left[3\left(\frac{8}{5}\right)+8\right] = \sum_{i=1}^{8} 3\left(\frac{i}{5}+i\right)$

Ex. 6. $4+(-16)+64+(-256)+\dots = \sum_{n=1}^{\infty} (-1)^{n-1}(4)^n$

$4^1 \quad -4^2 \quad 4^3 \quad -4^4$

Ex. 7. $10+10+10+10+10 = \sum_{k=1}^{5} 10$

NOW - Do you remember... ! - factorial

Ex. 8. $5! = 5\cdot4\cdot3\cdot2\cdot1 = 120$

on calculator: 5! ← math PRB #4

Simplify the following expressions.

Ex. 9. $\frac{5!}{7!} = \boxed{\frac{1}{42}}$

Ex. 10. $\frac{98!}{100!} = \frac{\cancel{98}\cdot\cancel{97}\cdot\cancel{96}\cdots\cancel{1}}{100\cdot99\cdot\cancel{98}\cdot\cancel{97}\cdot\cancel{96}\cdots\cancel{1}} = \boxed{\frac{1}{9900}}$

overflow in calculator!

Ex. 11. $\frac{(n)!}{(n+1)!} = \frac{\cancel{n}\cancel{(n-1)}\cancel{(n-2)}\cdots\cancel{1}}{(n+1)\cancel{(n)}\cancel{(n-1)}\cancel{(n-2)}\cdots\cancel{1}}$ $\boxed{\frac{1}{n+1}}$

Ex. 12. $\frac{(n-2)!}{n!} = \frac{\cancel{(n-2)}\cancel{(n-3)}\cdots\cancel{1}}{n(n-1)\cancel{(n-2)}\cancel{(n-3)}\cdots\cancel{1}} = \frac{1}{n(n-1)}$ or $\boxed{\frac{1}{n^2-n}}$

Using this knowledge, write the first 5 terms of the following sequence:

Ex. 13. $a_n = \frac{2^n}{n!}$ $\frac{2}{1}$, $\frac{4}{2}$, $\frac{8}{6}$, $\frac{16}{24}$, $\frac{32}{120}$

2, 2, $\frac{4}{3}$, $\frac{2}{3}$, $\frac{4}{15}$

Could you use "table" on your calculator here? yes ☺

For the following series

(a) write it in sigma notation

(b) find the indicated PARTIAL SUM - sum of part of the series

Ex. 14 $\frac{1}{2}+\frac{2}{4}+\frac{6}{8}+\frac{24}{16}+\frac{120}{32}+\dots = \sum_{i=1}^{\infty} \frac{i!}{2^i}$ (a) AND $S_3 = \frac{7}{4}$ (b)

$1\cdot2\cdot3 \quad 2\cdot3\cdot4 \quad 2\cdot3\cdot4\cdot5 \quad 2\cdot3\cdot4\cdot5\cdot6$

$2^1 \quad 2^2 \quad 2^3 \quad 2^4 \quad 2^5$