Pre-Calculus Notes

Name: Key

Section 9.2 – Arithmetic Sequences and Partial Sums

ARITHMETIC SEQUENCE: sequence in which each term is derived by adding the same number (common difference) to previous term

Common Difference: the difference between 2 consecutive terms

Today we are going to look for a "simple" way to find an explicit formula for an arithmetic sequence. Consider the following sequence. Do you see a pattern for obtaining each term?

a_1	a_2	a_3	a_4	a_5
5	8	11	14	17
5	(5+3)	(5+3)+3	((5+3)+3)+3	(((5+3)+3)+3)+3
5+(0)3	5+(1)3	5+(2)3	5+(3)3	5+(4)3

Therefore the *EXPLICIT FORMULA* is: $a_n = 5 + (n-1)3$

What is the *RECURSIVE FORMULA?* $a_1 = 5, \quad a_n = a_{n-1} + 3$

OR $a_{n+1} = a_n + 3$

General Form of an Arithmetic Sequence:

Recursive: $a_1,\ a_{n+1} = a_n + d$ **Explicit:** $a_n = a_1 + (n-1)d$

Example 1: For the following arithmetic sequences, find both the explicit and recursive formulas

Sequence	Recursive Formula AND Explicit Formula
a) 1, 9, 17, 25, 33, 41, 49, ... +8 +8 +8 +8 +8 +8	$a_1 =$ 1 $a_{n+1} =$ $a_n + 8$ $a_n =$ $1 + (n-1)8$ or $8n - 7$
b) 10, 7, 4, 1, -2, -5, ... −3 −3 −3 −3 −3	$a_1 = 10$ $a_{n+1} = a_n - 3$ $a_n = 10 + (n-1)(-3)$ or $a_n = -3n + 13$
c) a_1=10, d=6	$a_1 = 10$ $a_{n+1} = a_n + 6$ $a_n = 10 + (n-1)6$ or $a_n = 6n + 4$

c) $a_1=1$, $d=\frac{2}{5}$	$a_1 = 1$ $a_{n+1} = a_n + \frac{2}{5}$ $a_n = 1 + (n-1)\frac{2}{5}$ or $\frac{2}{5}n + \frac{3}{5}$

Can anyone see a "quicker" way to get the simplified version of the formula?

remember $y = mx + b$, fill in the 1st pairing and get b

$a_1 =$ d(1) + ___?___ $\Rightarrow a_n = d(n) + b$

Example 2: Find the explicit formula and the indicated term.

a) $a_4=1$, $a_{10}=37$ $a_n = -17 + (n-1)6$ $a_{21} = -17 + (21-1)6$

$\boxed{a_{21} = 103}$

$a_{10} = a_4 + (10-4)d$
$37 = 1 + 6d$
$36 = 6d$
$d = 6$

$a_4 = a_1 + (4-1)d$
$1 = a_1 + 3 \cdot 6$
$-17 = a_1$

Can you find the 21^{st} term <u>without</u> finding the general term formula? How? yes

$a_{21} = a_4 + (21-4)d$ or $a_{21} = a_{10} + (21-10)d$

$a_{21} = 1 + 17 \cdot 6$ $a_{21} = 37 + 11 \cdot 6$

$a_{21} = 103$ $a_{21} = 103$

b) $a_4=20$, $a_{13}=65$ $a_n = 5 + (n-1)5$ $a_{21} = 5 + (21-1)5$

$\boxed{a_{21} = 105}$

$a_{13} = a_4 + (13-4)d$
$65 = 20 + 9d$
$45 = 9d$
$d = 5$

$a_4 = a_1 + (4-1)d$
$20 = a_1 + 3 \cdot 5$
$5 = a_1$

Could you use "table" on your calculator to help you find the 21^{st} term and check your formula? yes

Write the first five terms of the sequence, find the common difference and write the nth term of the sequence as a function of n. (explicit formula)

c) $a_1=10$, $a_{k+1} = a_k + \frac{1}{2}$ 10 10.5 11 11.5 12

d = $\frac{1}{2}$ or 0.5 $a_n = 10 + (n-1)\frac{1}{2}$

OR $a_n = \frac{1}{2}n + 9.5$

d) $a_1=72$, $a_{k+1} = a_k - 6$ 72 66 60 54 48

d = -6 $a_n = 72 + (n-1)(-6)$

OR $a_n = -6n + 78$

Do you remember <u>*sigma notation?*</u> yes!

Example 1: Find the partial sum S_6 for the series.

a) $\sum_{n=1}^{\infty} 5n = 5+10+15+20+25+30+\ldots$ $5+10+15+20+25+30$ $S_6 =$ <u>105</u>	b) $\sum_{k=1}^{\infty}(10k-6) = 4+14+24+34+\ldots$ $4+14+24+34+44+54$ $S_6 =$ <u>174</u>

Just finding the terms and adding them up is good for series with a small number of terms. This is not a good method, however, if we have a large number of terms. *Let us see if we can find a short cut!*

Make up an arithmetic series with "say" six terms. Then write it backwards under the original. What do you notice?

3	7	11	15	19	23
23	19	15	11	7	3
26	26	26	26	26	26

wow!

* so sum is $\frac{6(26)}{2}$ because we had 2 series ⟹ would be average

$a_1 + (n-1)d$

ARITHMETIC SERIES: $S_n = \frac{n(a_1 + a_n)}{2}$ OR $S_n = \frac{n(2a_1 + (n-1)d)}{2}$

Example 2: Find each indicated partial sum.

a) S_{10} for $\sum_{k=1}^{\infty}(3k+1)$ $S_{10} = \frac{10(a_1 + a_{10})}{2}$ $S_{10} = \frac{10\cdot(4+31)}{2}$ $\boxed{S_{10} = 175}$	b) S_{34} for $12+7+2+(-3)\ldots$ $-5 \quad -5 \quad -5$ $d=-5$ $S_{34} = \frac{34(2\cdot 12 + (34-1)(-5))}{2}$ $\boxed{S_{34} = -2397}$