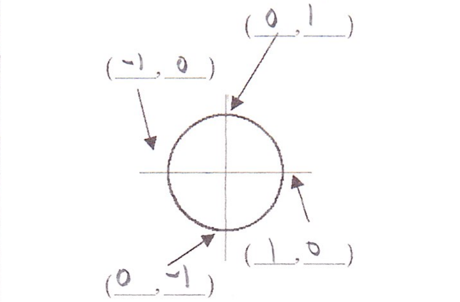
Pre-Calculus Notes

Name: Key

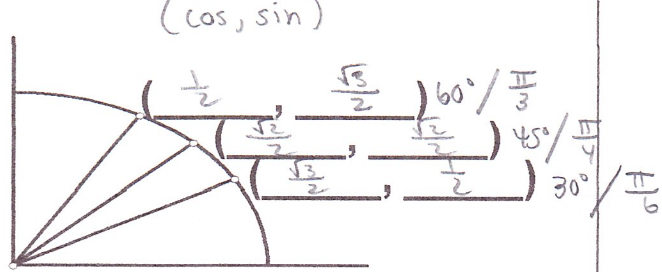
Section 5.3 - Solving Trigonometric Equations

QUADRANTAL ANGLES



QUADRANT I OF UNIT CIRCLE

(cos, sin)



DAY ONE Examples: Solve the following algebraically (no calculator). Then verify your answers with a calculator. **SHOW YOUR WORK.**

Algebraically Solved	Solution(s) Verified with Calculator

Algebraically Solved

1. $\sin x = \frac{+1}{2}$

$x = \frac{\pi}{6} + 2n\pi,\ n \in Z$

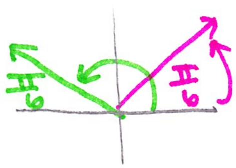
OR

$x = \frac{5\pi}{6} + 2n\pi,\ n \in Z$

S | A
T | C

Q I & II

ref ∠ $\frac{\pi}{6}$



Solution(s) Verified with Calculator

1. We could use a couple of techniques for checking our solution. * radian mode

Window:

$x_{min} = 0$ $x_{max} = 2\pi$ $x_{scl} = \frac{\pi}{6}$

$y_{min} = -2$ $y_{max} = 2$ $y_{scl} = \frac{1}{2}$

calc. a) **Intersect:** $y_1 = \sin x$

OR $y_2 = \frac{1}{2}$

b) **Zero:** $y_1 = \sin x - \frac{1}{2}$ (You MUST set the equation = to 0)

* could also calc. value

Algebraically Solved

2. $\sin x + \sqrt{2} = -\sin x$ **HINT:** Collect like terms.

$x = \frac{5\pi}{4} + 2n\pi,\ n \in Z$

OR

$x = \frac{7\pi}{4} + 2n\pi,\ n \in Z$

$2\sin x + \sqrt{2} = 0$

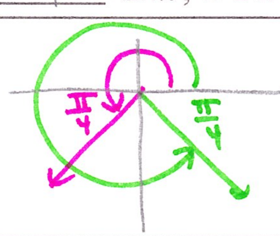
$2\sin x = -\sqrt{2}$

$\sin x = -\frac{\sqrt{2}}{2}$

S | A
T | C

Q III & IV

ref ∠ $\frac{\pi}{4}$



Solution(s) Verified with Calculator

2. Now check your answer by graphing the function and using one of the techniques seen in Example 1.

$\sin x + \sqrt{2} = -\sin x$

calc. intersection ✓

Algebraically Solved

3. $\tan x \cos^2 x = 2\tan x$

HINT: Set equal to zero and factor out a GCF.

$x = 0 + n\pi,\ n \in Z$

$\tan x \cos^2 x - 2\tan x = 0$

$\tan x(\cos^2 x - 2) = 0$

$\tan x = 0$ OR $\cancel{\cos^2 x - 2 = 0}$

$\frac{y}{x} = \frac{0}{1}$ $y = 0$ $\cancel{\cos^2 x = 2}$

$\cancel{\cos x = \pm\sqrt{2}}$

$x = 0 + 2n\pi$ and

$x = \pi + 2n\pi$

x-axis

not in domain of $\cos^{-1}(x)$!

Solution(s) Verified with Calculator

3. Again, it is good practice to verify your solutions! ☺

$\tan x(\cos x)^2 = 2\tan x$

calc. intersection ✓

4. $3\tan^2 x - 1 = 0$ over the interval $[0, 2\pi)$

HINT: Solve for $\tan^2 x$ and take the square root of both sides.

x = $\frac{\pi}{6}$, x = $\frac{5\pi}{6}$,

x = $\frac{7\pi}{6}$, or x = $\frac{11\pi}{6}$

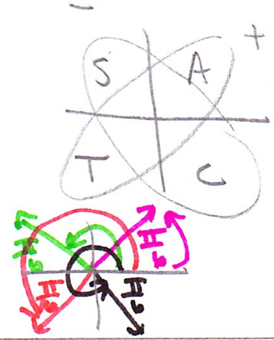
$3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\sqrt{\frac{1}{3}}$

$\tan x = \pm\frac{\sqrt{3}}{3}$

ref $\angle \frac{\pi}{6}$



4. Now ***ALGEBRAICALLY*** verify that your answers are correct.

$3\left(\tan\frac{\pi}{6}\right)^2 - 1 = 3\left(\frac{\sqrt{3}}{3}\right)^2 - 1$

$= 3 \cdot \frac{3}{9} - 1$

$= 1 - 1$

$= 0$ ✓

$3\left(\tan\frac{5\pi}{6}\right)^2 - 1 = 3\left(-\frac{\sqrt{3}}{3}\right)^2 - 1 = 0$ ✓

$3\left(\tan\frac{7\pi}{6}\right)^2 - 1 = 3\left(\frac{\sqrt{3}}{3}\right)^2 - 1 = 0$ ✓

$3\left(\tan\frac{11\pi}{6}\right)^2 - 1 = 3\left(-\frac{\sqrt{3}}{3}\right)^2 - 1 = 0$ ✓

5. $2\sin^2 x - \sin x - 1 = 0$ over the interval $[0, 2\pi)$

HINT: Factor as a quadratic or use the quadratic formula.

x = $\frac{7\pi}{6}$, x = $\frac{11\pi}{6}$,

or x = $\frac{\pi}{2}$

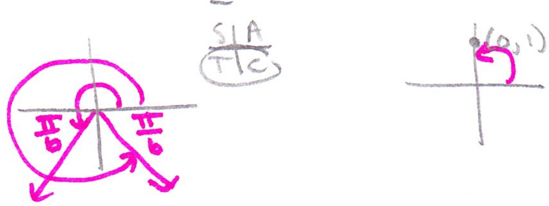
$(2\sin x + 1)(\sin x - 1) = 0$

$2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$

$\sin x - 1 = 0$

$\sin x = 1$



5. Again, check your answer! Let's not make silly mistakes. ☺

$2(\sin x)^2 - \sin x - 1 = 0$

calc. intersection

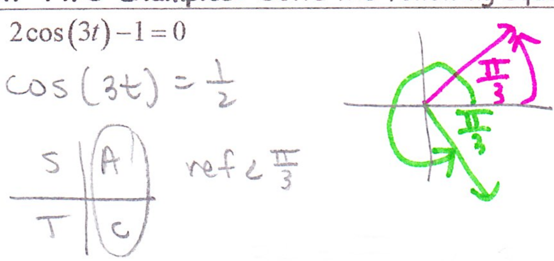
✓

DAY TWO Examples: Solve the following equations involving ***MULTIPLE angles***. $* n \in \mathbb{Z}$

1. $2\cos(3t) - 1 = 0$

$\cos(3t) = \frac{1}{2}$

ref $\angle \frac{\pi}{3}$



$3t = \frac{\pi}{3} + 2\pi n$ or $3t = \frac{5\pi}{3} + 2\pi n$,

therefore...

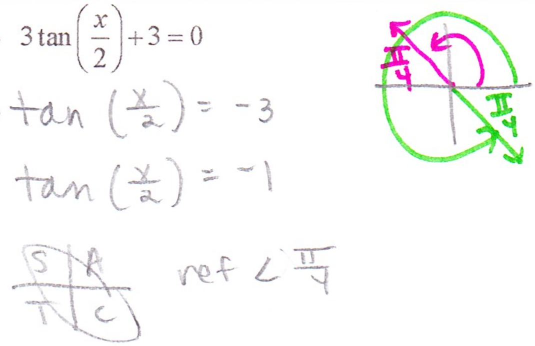
$t = \frac{\pi}{9} + \frac{2\pi}{3}n$ or $t = \frac{5\pi}{9} + \frac{2\pi}{3}n$.

2. $3\tan\left(\frac{x}{2}\right) + 3 = 0$

$3\tan\left(\frac{x}{2}\right) = -3$

$\tan\left(\frac{x}{2}\right) = -1$

ref $\angle \frac{\pi}{4}$



$\frac{x}{2} = \frac{3\pi}{4} + \pi n$ therefore

$x = \frac{3\pi}{2} + 2\pi n$

3. $2\sin^2 x + 3\cos x - 3 = 0$

HINT: Solve by rewriting & eliminating a function.

$2(1-\cos^2 x) + 3\cos x - 3 = 0$

$2 - 2\cos^2 x + 3\cos x - 3 = 0$

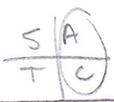
$-2\cos^2 x + 3\cos x - 1 = 0$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$2\cos x - 1 = 0$ $\quad\quad$ $\cos x - 1 = 0$

$\cos x = \frac{1}{2}$ $\quad\quad$ $\cos x = 1$



ref $\angle \frac{\pi}{3}$

$\boxed{x = \frac{\pi}{3} + 2\pi n,\ x = \frac{5\pi}{3} + 2\pi n, \text{ OR } x = 0 + 2\pi n}$

4. $\sec^2 x - 2\tan x = 4$

HINT: Solve by using the "inverse" function.

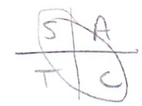
$1 + \tan^2 x - 2\tan x = 4$

$\tan^2 x - 2\tan x - 3 = 0$

$(\tan x - 3)(\tan x + 1) = 0$

$\tan x = 3$ $\quad\quad$ $\tan x = -1$

$x = \arctan 3$



ref $\angle \frac{\pi}{4}$

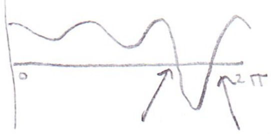
$\boxed{x = \arctan 3 + \pi n \text{ OR } x = \frac{3\pi}{4} + \pi n}$

5. $4\sin^3 x - 2\sin^2 x - 2\sin x + 2 = 0$

Solve by using a calculator. Approximate solutions to 3 decimal places over the interval $[0, 2\pi)$.

$y_1 = 4(\sin x)^3 - 2(\sin x)^2 - 2\sin x + 2$

$y_2 = 0$



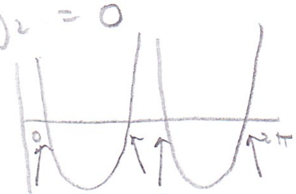
$\boxed{x \approx 4.120 \text{ rads OR } 5.305 \text{ rads}}$

6. $\csc^2 x + 0.5\cot x - 5 = 0$

Solve by using a calculator. Approximate solutions to 3 decimal places over the interval $[0, 2\pi)$.

$y_1 = \left(\frac{1}{\sin x}\right)^2 + \frac{0.5}{\tan x} - 5$

$y_2 = 0$



$\boxed{x \approx 0.515 \text{ rads}, 2.726 \text{ rads}, 3.657 \text{ rads, OR } 5.868 \text{ rads}}$

4 sol'ns!

program in calculator!

DAY THREE Examples:

Solve the following by using the quadratic formula, then graph and check your solutions. $*n \in \mathbb{Z}$

1. $3\tan^2 x + 4\tan x - 4 = 0$

$\underset{a}{3}\ \underset{b}{4}\ \underset{c}{-4}$

$\tan x = \frac{-4 \pm \sqrt{16 - 4(-12)}}{6}$

$\tan x = \frac{2}{3}$ OR $\tan x = -2$

$x = \tan^{-1}\left(\frac{2}{3}\right)$ OR $x = \tan^{-1}(-2)$

$\boxed{x \approx 0.588 + \pi n \text{ OR } -1.107 + \pi n}$

2. $\csc^2 x + 0.5\cot x - 5 = 0$

$1 + \cot^2 x + 0.5\cot x - 5 = 0$

$\underset{a}{1}\cot^2 x + \underset{b}{0.5}\cot x \underset{c}{-4} = 0$

$\cot x = \frac{-0.5 \pm \sqrt{.25 - 4(-4)}}{2}$

$\cot x \approx 1.766$ OR $\cot x \approx -2.266$

$x \approx \tan^{-1}\left(\frac{1}{1.766}\right)$ OR $x \approx \tan^{-1}\left(\frac{1}{-2.266}\right)$

$\boxed{x \approx 0.515 + \pi n \text{ OR } -0.416 + \pi n}$

WORD PROBLEMS:

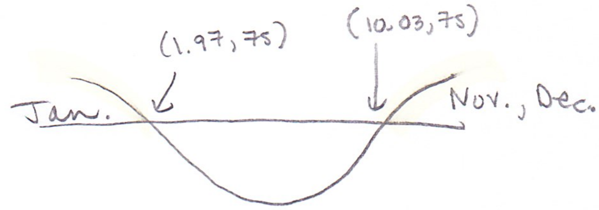
3. The monthly sales S (in hundreds of units) of skiing equipment are approximated by $S = 58.3 + 32.5\cos\left(\frac{\pi t}{6}\right)$ where t is the time (in months), with t = 1 corresponding to January. Determine the months when the sales exceed 7500 units.

(above $y = 75$)

$S = \frac{7500}{100}$

$S = 75$

$$\underset{y_1}{75} = \underset{y_2}{58.3 + 32.5\cos\left(\frac{\pi t}{6}\right)}$$



Calculate intersection

$x_{min} = 0$ $\quad x_{max} = 12$ (12 months in year)

$y_{min} = 0$ $\quad y_{max} = 150$ (need to be able to see 75)

$$\boxed{\text{January, November, December}}$$

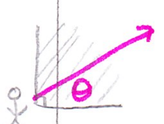
4. A sharpshooter intends to hit a target at a distance of 1000 yds. With a gun that has a muzzle velocity of 1200 ft. per second. Neglecting air resistance, determine the gun's minimum angle of elevation θ if the range is given by the function $r = \frac{1}{32}v_0^2(\sin 2\theta)$.

1000 yds = 3000 ft

$$\underset{y_1}{3000} = \underbrace{\frac{1}{32}(1200)^2(\sin 2\theta)}_{y_2}$$

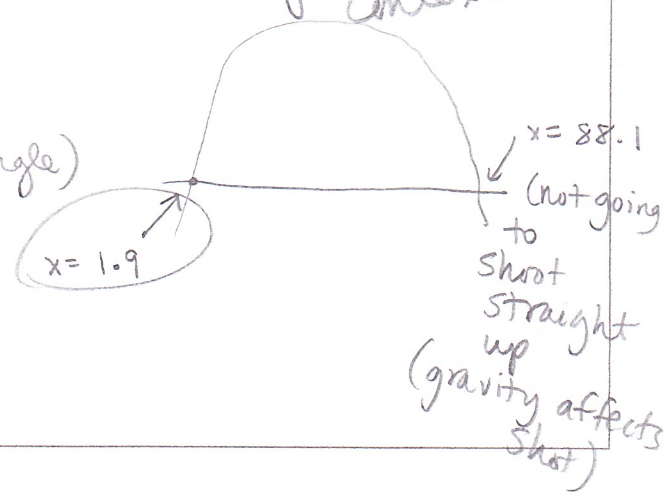
* do degrees → makes more sense w/ problem context

Calculate intersection



$x_{min} = 0$ $\quad x_{max} = 90$ (x = angle)

$y_{min} = 0$ $\quad y_{max} = 5000$



$$\boxed{1.9°}$$