Pre-Calculus Notes   Name: Key

Section 5.4 - Sum and Difference Formulas

Example 1: True or False? Try it out with your calculator.

a. $\cos 55° = \cos 20° + \cos 35°$

$0.5736 \stackrel{?}{=} 0.9397 + 0.8192$

$0.5736 \neq 1.7589$

What a bummer!

b. $\sin 20° = \sin(90° - 70°)$

$0.3420 = 0.3420$ ✓

Note: We will generally use the following formulas to find ***EXACT*** function values for angles other than $30°, 45°, or\ 60°$.

Refer to your formula sheet for the Sum and Difference Formulas. Mrs. Pesek will teach you a song to help you remember the sum and difference formulas of sine and cosine.

Example 2: Express each angle in terms of the sine, cosine, or tangent of one angle.

a. $\cos 20° \cos 35° - \sin 20° \sin 35°$

$\cos(20° + 35°)$

$\boxed{\cos 55°}$

b. $\cos\frac{7\pi}{6}\cos\frac{5\pi}{6} + \sin\frac{7\pi}{6}\cdot\sin\frac{5\pi}{6}$

$\cos\left(\frac{7\pi}{6} - \frac{5\pi}{6}\right)$

$\cos\frac{2\pi}{6}$

$\boxed{\cos\frac{\pi}{3}}$

c. $\sin 40° \cos 35° - \cos 40° \sin 35°$

$\sin(40° - 35°)$

$\boxed{\sin 5°}$

d. $\dfrac{\tan 140° - \tan 60°}{1 + (\tan 140°)(\tan 60°)}$

$\tan(140° - 60°)$

$\boxed{\tan 80°}$

Example 3: Use the identities to find the EXACT value of the expression

a. $\cos 15° \cos 60° + \sin 15° \sin 60°$

$\cos(15° - 60°)$

$\cos(-45°) = \boxed{+\frac{\sqrt{2}}{2}}$

S | A

T | C

Q IV

ref ∠ 45°

b. $\dfrac{\tan\left(\frac{5\pi}{4}\right) - \tan\left(\frac{\pi}{12}\right)}{1 + \tan\left(\frac{5\pi}{4}\right)\tan\left(\frac{\pi}{12}\right)} = \tan\left(\frac{5\pi}{4} - \frac{\pi}{12}\right)$

$= \tan\frac{7\pi}{6}$

S | A

T | C

Q III

ref ∠ $\frac{\pi}{6}$

$= \boxed{+\frac{\sqrt{3}}{3}}$

Use the formulas above to find the ***EXACT VALUES*** for the following functions:

Ex. 4) $\cos(75°) = \cos(\underline{45°} + \underline{30°}) = \cos 45° \cos 30° - \sin 45° \sin 30°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

Ex. 5) $\sin\left(\overset{75°}{\frac{5\pi}{12}}\right) = \sin\left(\overset{30°}{\frac{\pi}{6}} + \overset{45°}{\frac{\pi}{4}}\right) = \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \cos\frac{\pi}{6}\sin\frac{\pi}{4}$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

QII ref∠30°

*either will work, and more ways aren't listed

Ex.6) $\sin(195°) = \sin(\underbrace{\underline{45°}}_{QI} + \underbrace{\underline{150°}}_{QII}) \; or \; \sin(\underline{240°} - \underline{45°})$

$= \sin 45° \cos 150° + \cos 45° \sin 150°$

$= \frac{\sqrt{2}}{2} \cdot -\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{-\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \boxed{\frac{-\sqrt{6}+\sqrt{2}}{4} \text{ OR } \frac{\sqrt{2}-\sqrt{6}}{4}}$

Ex. 7) $\sin\left(\overset{-105°}{\cancel{\frac{-7\pi}{12}}}\right) = \underline{\sin(45 - 150°)}$ QII ref∠30°

$= \sin 45° \cos 150° - \cos 45° \sin 150°$

$= \frac{\sqrt{2}}{2} \cdot -\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$

*avoid 30° w/ tangent if you want to make your life easier!

Ex. 8) $\tan\left(\frac{-7\pi}{12}\right)$ ($-105°$) $= \tan(-45° - 60°)$

$= \dfrac{\tan(-45°) - \tan 60°}{1 + \tan(-45°)(\tan 60°)}$

* $-45°$ is in QIV

$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$

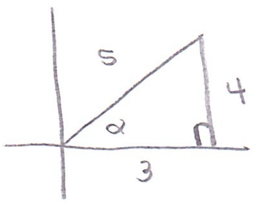
$= \dfrac{-1 - \sqrt{3}}{1 + (-1)\cdot\sqrt{3}}$

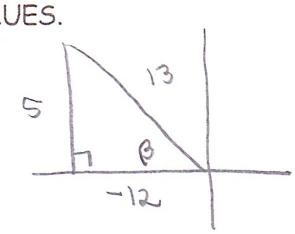
*have to rationalize the denom.

$= \dfrac{(-1-\sqrt{3})}{(1-\sqrt{3})} \cdot \dfrac{(1+\sqrt{3})}{(1+\sqrt{3})} = \dfrac{-1-2\sqrt{3}-3}{1-3} = \dfrac{-4-2\sqrt{3}}{-2} = \boxed{2+\sqrt{3}}$

Example 4: Given $0 < \alpha < \frac{\pi}{2}$ with $\cos\alpha = \frac{3}{5}$, and $\frac{\pi}{2} < \beta < \pi$ with $\sin\beta = \frac{5}{13}$.

Find the following EXACT VALUES.





<table>
<tr><td>$\sin\alpha$ $\frac{4}{5}$</td><td>$\cos\beta$ $\frac{-12}{13}$</td><td>$\tan\alpha$ $\frac{4}{3}$</td><td>$\tan\beta$ $-\frac{5}{12}$</td></tr>
<tr><td colspan="2">$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$= \frac{3}{5}\cdot\frac{-12}{13} - \frac{4}{5}\cdot\frac{5}{13}$
$= \boxed{-\frac{56}{65}}$</td><td colspan="2">$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
$= \frac{3}{5}\cdot\frac{-12}{3} + \frac{4}{5}\cdot\frac{5}{13}$
$= \boxed{-\frac{16}{65}}$</td></tr>
<tr><td colspan="2">$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
$= \frac{4}{5}\cdot\frac{-12}{13} - \frac{3}{5}\cdot\frac{5}{13}$
$= \boxed{\frac{-63}{65}}$</td><td colspan="2">$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$= \frac{4}{5}\cdot\frac{-12}{13} + \frac{3}{5}\cdot\frac{5}{13}$
$= \boxed{\frac{-33}{65}}$</td></tr>
<tr><td colspan="2">$\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$
$= \dfrac{\left(\frac{4}{3} - \frac{-5}{12}\right)}{\left(1 + \frac{4}{3}\cdot\frac{-5}{12}\right)} = \boxed{\frac{63}{16}}$</td><td colspan="2">$\sec(\alpha+\beta) = \dfrac{1}{\cos(\alpha+\beta)}$
$= \dfrac{1}{\frac{-56}{65}} = \boxed{\frac{-65}{56}}$</td></tr>
<tr><td colspan="2">$\cot(\alpha-\beta) = \dfrac{1}{\tan(\alpha-\beta)}$
$= \dfrac{1}{\frac{63}{16}} = \boxed{\frac{16}{63}}$</td><td colspan="2">$\csc(\alpha-\beta) = \dfrac{1}{\sin(\alpha-\beta)}$
$= \dfrac{1}{\frac{-63}{65}} = \boxed{\frac{-65}{63}}$</td></tr>
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