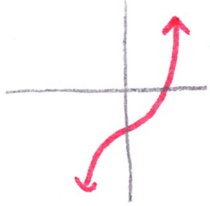
Pre-Calculus Fall Final Review

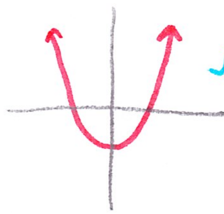
1st six weeks

A. $y = 2x^3 - 1$



passes VLT and HLT

B. $y = 3x^2 - 1$



passes VLT

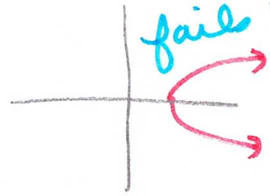
C. $x - 3y^2 = 7$

$-3y^2 = -x + 7$

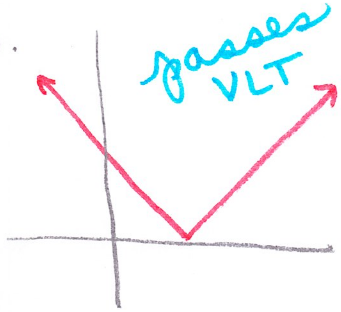
$y^2 = \frac{-x+7}{-3}$

$y = \pm\sqrt{\frac{x-7}{3}}$

fails VLT



D.



passes VLT

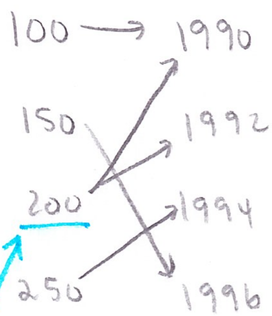
no x repeats

E.

x	0	1	3
y	-4	5	-4

y repeats

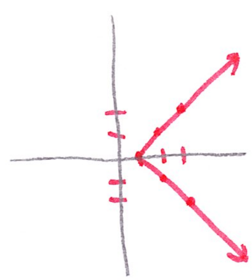
F.



goes to two different y-values, making x repeat

G. $x = |y| + 1$

x	y
3	-2
3	2
2	-1
2	1
1	0



fails VLT

① functions → pass VLT; no x repeats

A, B, D, E

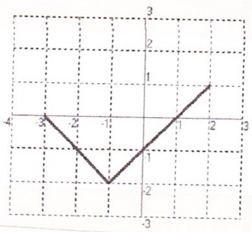
② NOT functions → fail VLT; x repeats

C, F, G

③ one-to-one → pass VLT & HLT; no x or y repeats

A

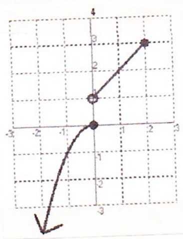
④



D: $[-3, 2]$

R: $[-2, 1]$

⑤



D: $(-\infty, 2]$

R: $(-\infty, 0] \cup (1, 3]$

⑥ $y = \sqrt{x-5}$

$x - 5 \geq 0$ $\quad y \geq 0$

$x \geq 5$

D: $[5, \infty)$ $\quad$ R: $[0, \infty)$

#7–10 see graph paper

$f(x) = |2x+3|$ $\quad g(x) = x^2 - x$ $\quad h(x) = 5x+1$

$t(x) = \begin{cases} 3x^2 - 1, & x > -1 \\ 4 - x, & x \leq -1 \end{cases}$ $\quad s(x) = [\![x-5]\!]$

⑪ $g(-2) = (-2)^2 - -2$

$= 4 + 2$

$= \boxed{6}$

⑫ $(f \circ g)(-1) = f(g(-1)$

$= f((-1)^2 - -1)$

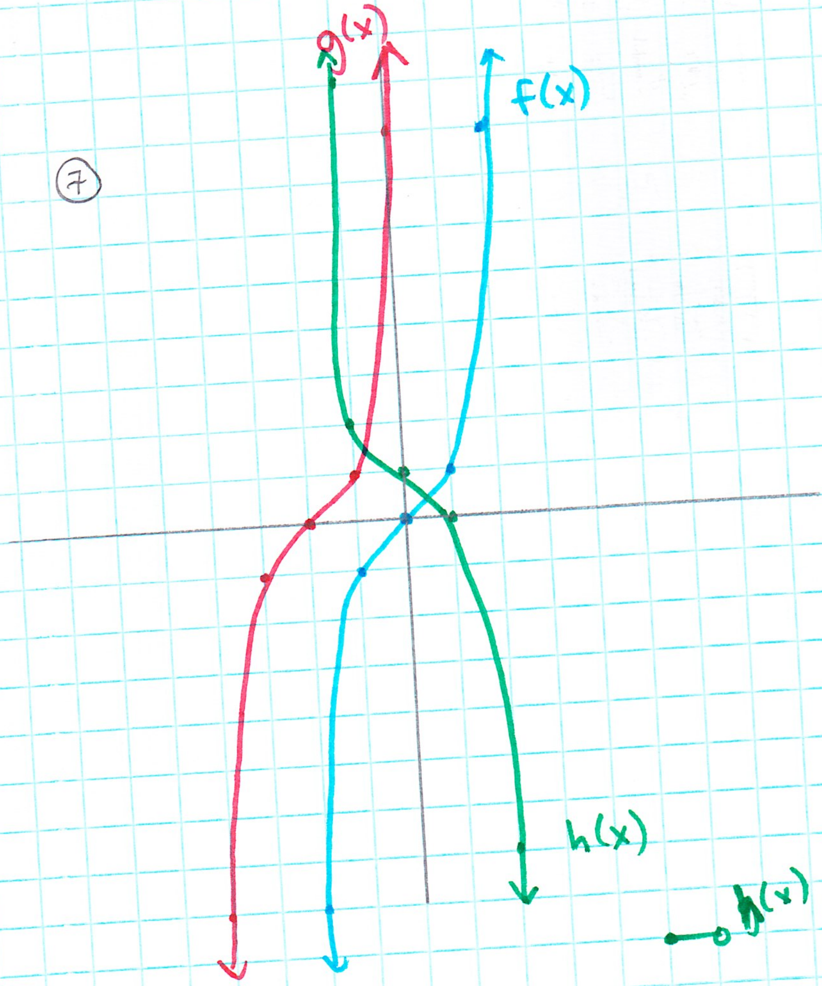
$= f(2)$

$= |2 \cdot 2 + 3|$

$= |7|$

$= \boxed{7}$

7



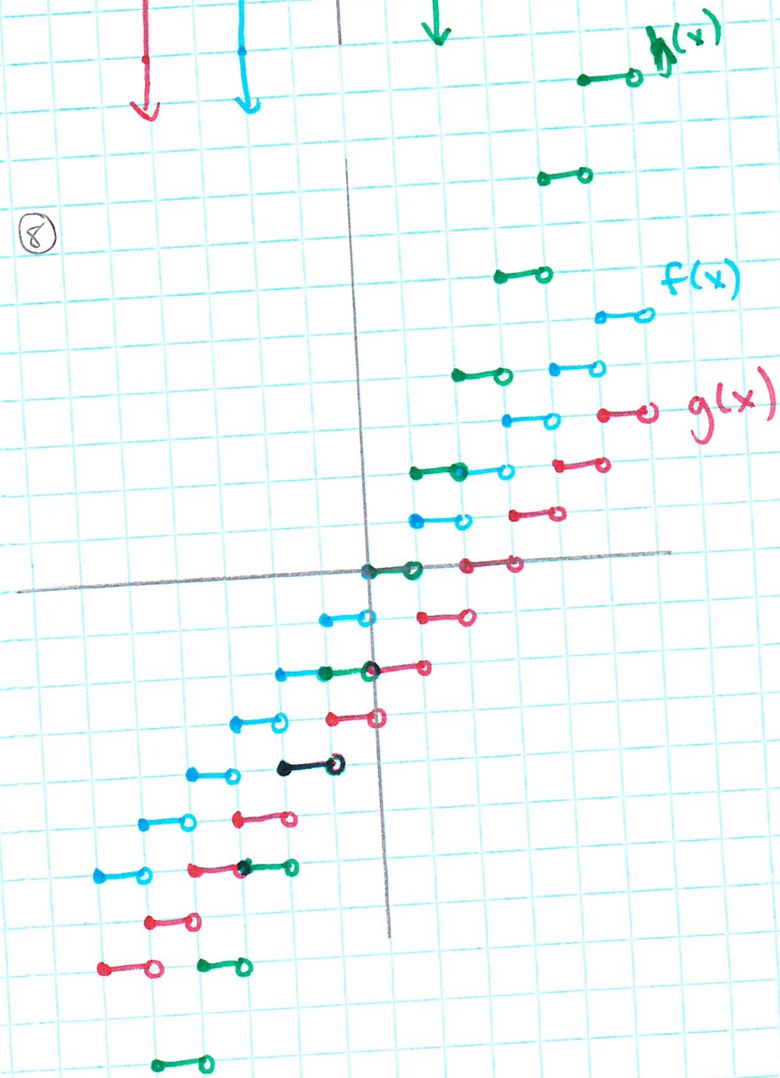
$g(x) = f(x+2)$

2 to the left

$h(x) = -f(x) + 1$

across x-axis and up 1

8



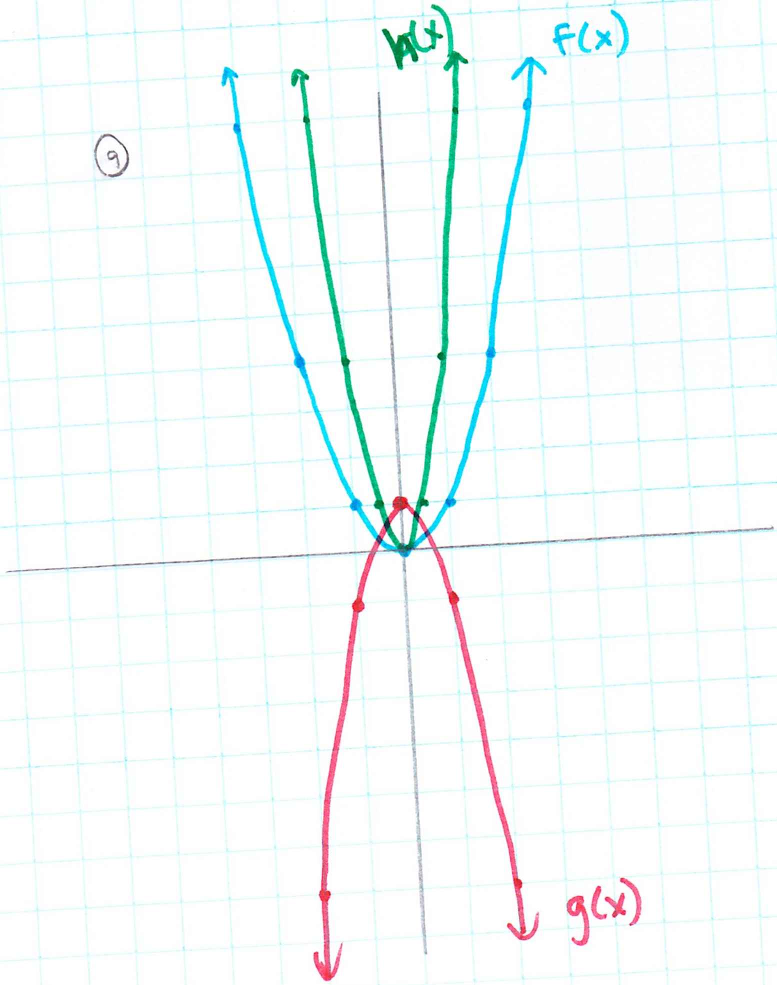
$g(x) = f(x) - 2$

down 2

$h(x) = 2f(x)$

vertical stretch factor of 2

9



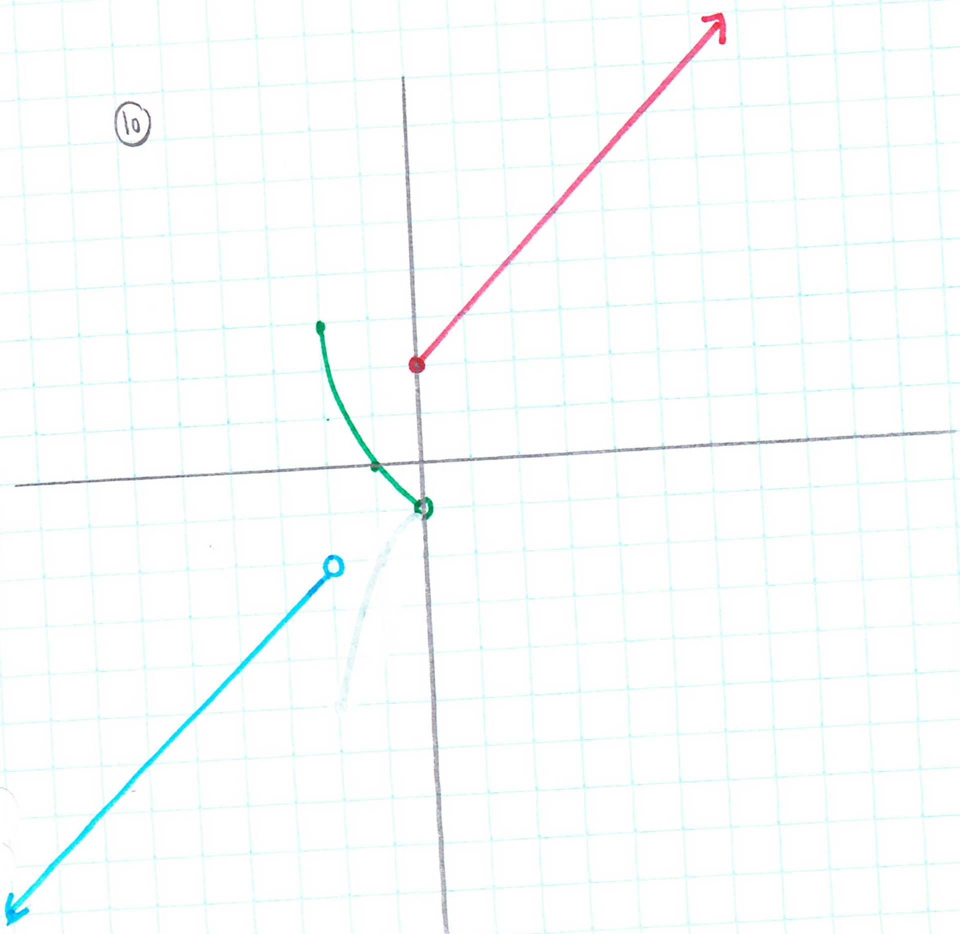
$g(x) = -2f(x) + 1$

across x-axis
vert. stretch factor 2
up 1

$h(x) = f(2x)$

horiz. compression
factor of $\frac{1}{2}$

10



(13) $(g-h)(x) = g(x) - h(x)$

$= x^2 - x - (5x+1)$

$= x^2 - x - 5x - 1$

$= \boxed{x^2 - 6x - 1}$

(14) $(h \circ g)(x) = h(g(x))$

$= h(x^2 - x)$

$= 5(x^2 - x) + 1$

$= \boxed{5x^2 - 5x + 1}$

(15) $t(-2)$

$-2 \leq -1$, so use $4 - x$

$t(-2) = 4 - -2$

$= \boxed{6}$

(16) $t(4)$

$4 > -1$, so use $3x^2 - 1$

$t(4) = 3 \cdot 4^2 - 1$

$= 3 \cdot 16 - 1$

$= 48 - 1$

$= \boxed{47}$

(17) $s(8.2) = [\![8.2 - 5]\!]$

$= [\![3.2]\!]$

= greatest integer less than or equal to 3.2

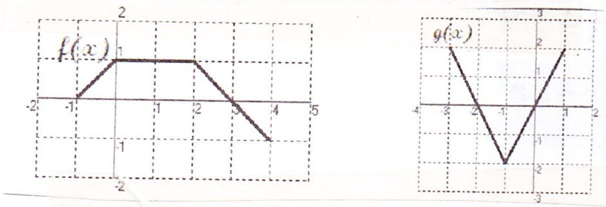
$= \boxed{3}$

(18) $s(1.5) = [\![1.5 - 5]\!]$

$= [\![-3.5]\!]$

$= \boxed{-4}$

(19)



ⓐ $(f+g)(1)$

$= f(1)+g(1)$

$= 1+2$

$= \boxed{3}$

ⓑ $(g-f)(0) = g(0)-f(0)$

$= 0-1$

$= \boxed{-1}$

ⓒ $(f \cdot g)(-1) = f(-1)\cdot g(-1)$

$= 0 \cdot -2$

$= \boxed{0}$

ⓓ $(f \circ g)(-2) = f(g(-2))$

$= f(0)$

$= \boxed{1}$

ⓔ $(g \circ f)(2) = g(f(2))$

$= g(1)$

$= \boxed{2}$

(20) Ⓐ $f(x) = 5-2x$

$y = 5-2x$

<u>inverse</u>

$x = 5-2y$

$x-5 = -2y$

$y = \frac{x-5}{-2}$

$$\boxed{f^{-1}(x) = -\frac{1}{2}x + \frac{5}{2}}$$

Ⓑ $f(x) = 2x^3-1$

$y = 2x^3-1$

<u>inverse</u>

$x = 2y^3-1$

$2y^3 = x+1$

$y^3 = \frac{x+1}{2}$

$y = \sqrt[3]{\frac{x+1}{2}}$

$$\boxed{f^{-1}(x) = \sqrt[3]{\frac{x+1}{2}}}$$

(21)

x	-3	0	2	5
y	3	-2	-5	-11

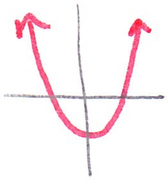
$f^{-1}(x)$ (switch x and y)

x	3	-2	-5	-11
y	-3	0	2	5

Domain: $\{-11, -5, -2, 3\}$

Range: $\{-3, 0, 2, 5\}$

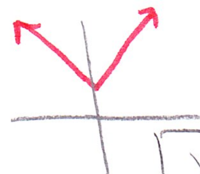
(22) (A) $f(x) = x^2 - 1$



$\boxed{\text{even}}$

$\boxed{\text{y-axis symm.}}$

(B) $f(x) = |x| + 1$



$\boxed{\text{even}}$

$\boxed{\text{y-axis symm.}}$

(C) $f(x) = 3x^3 - x$ ←

$f(-x) = 3(-x)^3 - (-x)$

$f(-x) = -3x^3 + x$ ←

opposite!

$\boxed{\text{odd}}$ $\boxed{\text{origin symm.}}$

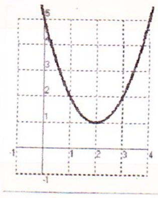
(D) $f(x) = x^5 - x^3 + 1$

$f(-x) = -x^5 + x^3 + 1$

$\boxed{\text{neither}}$

$\boxed{\text{no symm.}}$

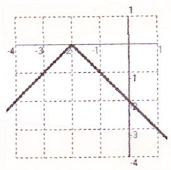
(23)



2 right
up 1
no stretch

$\boxed{y = (x-2)^2 + 1}$

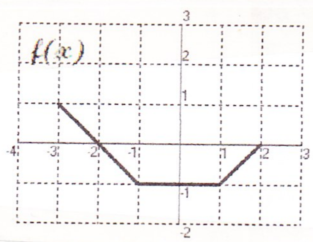
(24)



left 2
across x-axis
no stretch

$\boxed{y = -|x+2|}$

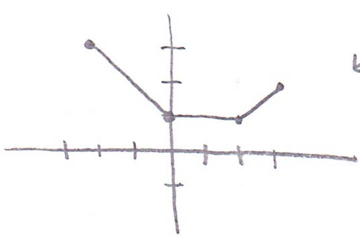
(25)



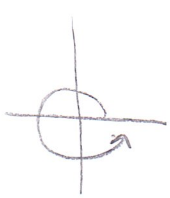
(A) increasing $(1, 2)$
decreasing $(-3, -1)$
constant $(-1, 1)$

(B) $g(x) = f(x-1) + 2$

one right and up 2



(26)



$\frac{5}{12}(+360)$

$\boxed{150°}$

(27) $-790° + 360° = -430°$

$-430° + 360° = -70°$

$-70° + 360° = 290°$

co-terminal

$\boxed{\text{Q IV}}$

(28) $1350° - 360° = 990°$

$990° - 360° = 630°$

$630° - 360° = 270°$

$\boxed{\text{neg y-axis}}$

(29) $\dfrac{10\pi}{3} - 2\pi = \dfrac{4\pi}{3}$

co-terminal

$\boxed{\text{Q III}}$

(30) $264°$

$\boxed{624°, -96°, \text{etc}}$

(31) $\dfrac{7\pi}{12}$

$\dfrac{7\pi}{12} + 2\pi = \dfrac{31\pi}{12}$,

$\dfrac{7\pi}{12} - 2\pi = \dfrac{-17\pi}{12}$

etc.

(32) $80° 25' 45''$

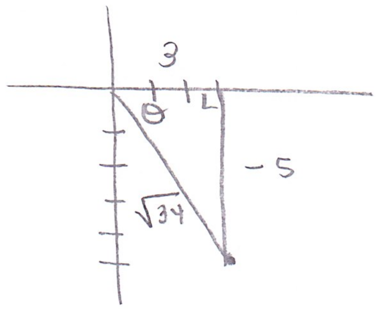
use calculator

$\boxed{\approx 80.43°}$

(33) $\dfrac{785°}{1} \cdot \dfrac{\pi}{180°}$

$\boxed{\dfrac{157\pi}{36}}$

(34) $\dfrac{11\pi}{9} \cdot \dfrac{180°}{\pi} = \boxed{220°}$



(35) $\sin\theta = \dfrac{-5}{\sqrt{34}}$

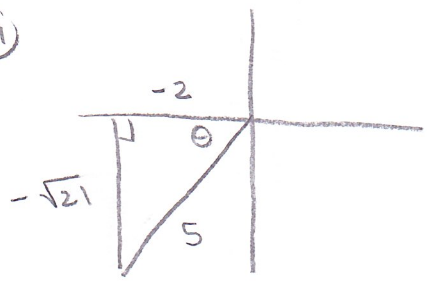
$\boxed{\sin\theta = \dfrac{-5\sqrt{34}}{34}}$

(36) $\boxed{\sec\theta = \dfrac{\sqrt{34}}{3}}$

(37) $\boxed{\tan\theta = -\dfrac{5}{3}}$

(38) $\boxed{\cot\theta = -\dfrac{3}{5}}$

(39)



$\sin\theta = \dfrac{-\sqrt{21}}{5}$

so $\csc\theta = \dfrac{-5}{\sqrt{21}}$

$\boxed{\csc\theta = \dfrac{-5\sqrt{21}}{21}}$

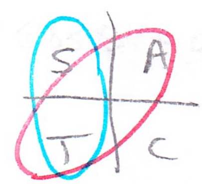
(40)

$\sin\theta > 0$

$\cot\theta < 0$

$\boxed{\text{II}}$

(41)

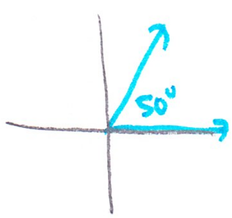


$\cos\theta < 0$

$\tan\theta > 0$

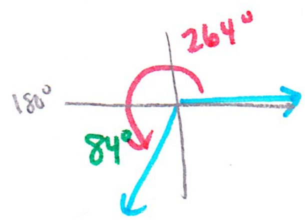
$\boxed{\text{III}}$

(42) (a) $-310° + 360° = 50°$



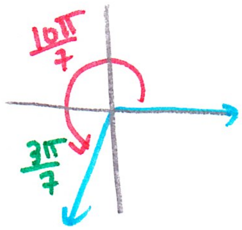
$\boxed{50°}$

(b) $264°$



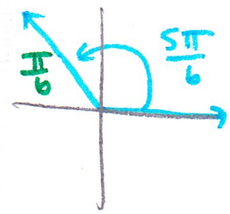
$\boxed{84°}$

(c) $\frac{10\pi}{7}$



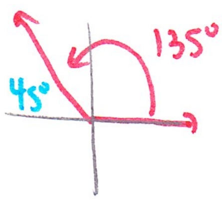
$\boxed{\frac{3\pi}{7}}$

(d) $\frac{5\pi}{6}$

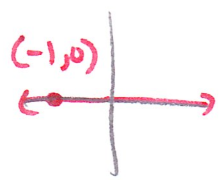


$\boxed{\frac{\pi}{6}}$

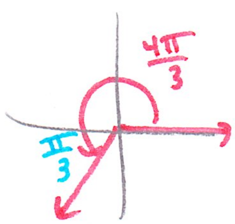
(43) $\sec 135^\circ = \boxed{-\sqrt{2}}$



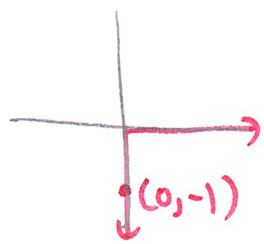
(44) $\tan \pi = \boxed{0}$



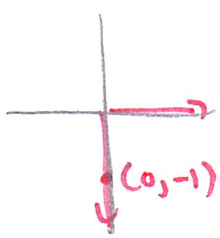
(45) $\tan \frac{4\pi}{3} = \boxed{\sqrt{3}}$



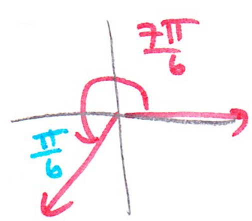
(46) $\sin(-90^\circ) = \boxed{-1}$



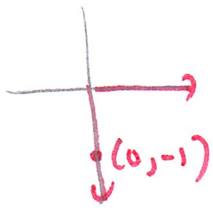
(47) $\csc 270^\circ = \boxed{-1}$



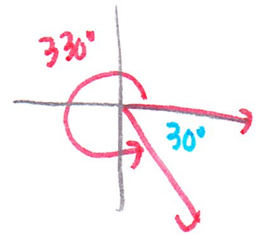
(48) $\cot \frac{7\pi}{6} = \boxed{\sqrt{3}}$



(49) $\cos \frac{3\pi}{2} = \boxed{0}$



(50) $\sin 330^\circ = \boxed{-\frac{1}{2}}$



(51) $\sec \frac{3\pi}{5} = \dfrac{1}{\cos \frac{3\pi}{5}}$

$\boxed{\approx -3.2361}$

(52) $\csc 27.8^\circ = \dfrac{1}{\sin 27.8^\circ}$

$\boxed{\approx 2.1441}$

(53) $\cot \frac{11\pi}{8} = \dfrac{1}{\tan \frac{11\pi}{8}}$

$\boxed{\approx 0.4142}$

$\boxed{2^{nd} \text{ six weeks}}$

(54) $y = -\cos\left(2x + \frac{\pi}{2}\right) + 3$

$y = -1\cos\left(2\left(x + \frac{\pi}{4}\right)\right) + 3$

(a) 1 (b) $\frac{2\pi}{2} = \pi$ (c) $\frac{\pi}{4}$ left (d) up 3

(e) no b/c shifted left (f) no

(55) $y = 2\sin 3\left(x - \frac{\pi}{2}\right) - 1$

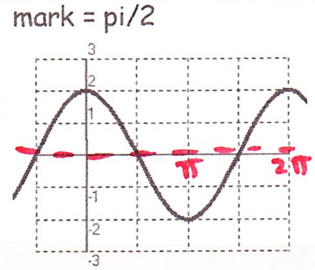
(a) 2 (b) $\frac{2\pi}{3}$ (c) down 1 (d) $\frac{\pi}{2}$ right

(56) $y = -3\tan(4x - \pi) + 2$

$y = -3\tan\left(4\left(x - \frac{\pi}{4}\right)\right) + 2$

(a) none (b) $\frac{\pi}{4}$ (c) up 2 (d) $\frac{\pi}{4}$ right

(57) each x-axis tick mark = pi/2



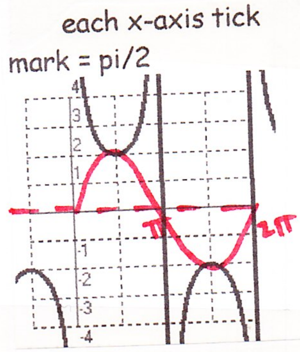
cosine $\rightarrow$ no flip

amp = 2

period = 2π

$\boxed{y = 2\cos x}$

(58)



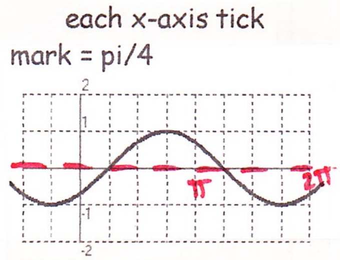
csc → no flip

"amp" = 2

period = 2π

$$\boxed{y = 2\csc x}$$

(59)



sine → no flip

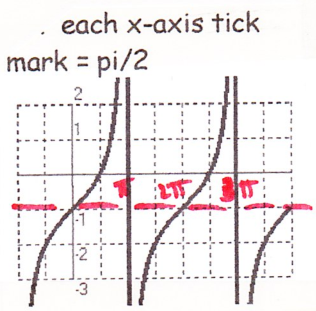
amp = 1

period = 2π

right $\frac{\pi}{4}$

$$\boxed{y = \sin\left(x - \frac{\pi}{4}\right)}$$

(60)



tangent → no flip

"amp" = 1

period = 2π

down 1

$$\boxed{y = \tan\left(\frac{1}{2}x\right) - 1}$$

(61) (a) sin 34°

$\cos(90-34) = \boxed{\cos 56°}$

(b) csc 75°

$\sec(90-75) = \boxed{\sec 15°}$

(c) $\tan\frac{\pi}{5}$

$\cot\left(\frac{\pi}{2} - \frac{\pi}{5}\right) = \boxed{\cot\frac{3\pi}{10}}$

(d) $\cos\frac{3\pi}{7}$

$\sin\left(\frac{\pi}{2} - \frac{3\pi}{7}\right) =$

$\boxed{\sin\frac{\pi}{14}}$

(62) $\tan\theta = 0.4378$

$\theta = \tan^{-1}(0.4378)$

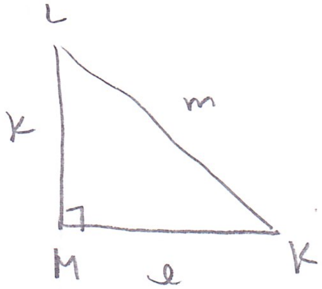
$\boxed{\theta \approx 24^\circ}$

(63) $\sec\theta = 2.1569$

$\cos\theta = \dfrac{1}{2.1569}$

$\theta = \cos^{-1}\left(\dfrac{1}{2.1569}\right)$

$\boxed{\theta \approx 62^\circ}$

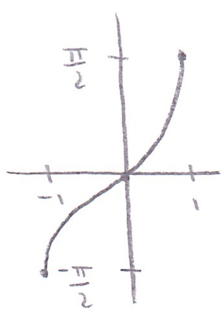


(64) $\sec K = \boxed{\dfrac{m}{l}}$

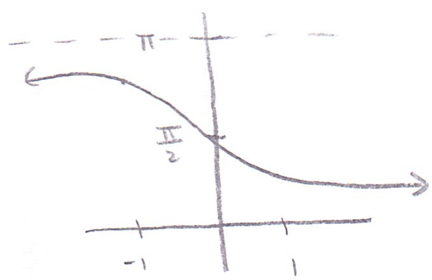
(65) $\tan L = \boxed{\dfrac{l}{k}}$

(66) $\cot K = \boxed{\dfrac{l}{k}}$

(67) $y = \sin^{-1} x$



(68) $y = \cot^{-1} x$



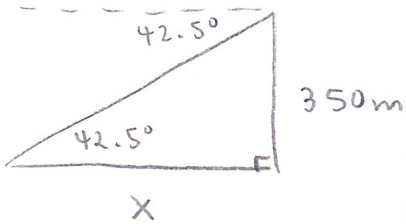
(69) (a) $\tan^{-1}(\sqrt{3}) = \boxed{\dfrac{\pi}{3}}$

(b) $\sin^{-1}\left(-\dfrac{\sqrt{2}}{2}\right) = \boxed{-\dfrac{\pi}{4}}$

(c) $\text{Arccot}(-1) = \boxed{135^\circ}$

(d) $\text{Arcsec}\left(\dfrac{2\sqrt{3}}{3}\right) = \boxed{\dfrac{\pi}{6}}$

(70)

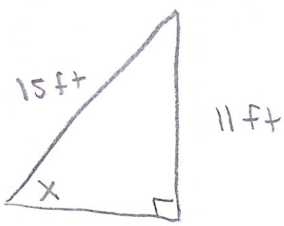


$$\tan 42.5° = \frac{350}{x}$$

$$x = \frac{350}{\tan 42.5°}$$

$$x \approx \boxed{382.0 \text{ m}}$$

(71)



$$\sin x = \frac{11}{15}$$

$$x = \sin^{-1}\left(\frac{11}{15}\right)$$

$$x \approx \boxed{47°}$$

(72) $\frac{\cos\theta}{\sin\theta} = \cot\theta$ $\boxed{D}$

(73) $\sin^2\theta = 1 - \cos^2\theta$ $\boxed{B}$

(74) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\boxed{E}$

(75) $\frac{1}{\cos^2\theta} = \sec^2\theta$ $\boxed{F}$

(76) $\frac{1}{\tan^2\theta} = \cot^2\theta$ $\boxed{C}$

(77) $\csc^2\theta = 1 + \cot^2\theta$ $\boxed{A}$

(78) $\sin\theta \cdot \sec\theta$

$\sin\theta \cdot \frac{1}{\cos\theta}$

$\frac{\sin\theta}{\cos\theta}$

$\tan\theta$

$\boxed{A}$

(79) $\tan\theta \cdot \csc\theta \cdot \cot\theta$

$\frac{\cancel{\sin\theta}}{\cancel{\cos\theta}} \cdot \frac{\csc\theta}{1} \cdot \frac{\cancel{\cos\theta}}{\cancel{\sin\theta}}$

$\csc\theta$

$\boxed{F}$

(80) $(1-\cos^2\theta)(-\csc^2\theta)$

$\frac{\sin^2\theta}{1} \cdot \frac{1}{-\sin^2\theta}$

-1

$\boxed{D}$

(81) $\sec\theta \cdot \frac{\sin\theta}{\cot\theta}$

$\frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\frac{\cos\theta}{\sin\theta}}$

$\frac{\sin\theta}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta}$

$\frac{\sin^2\theta}{\cos^2\theta}$

$\tan^2\theta$

$\boxed{G}$

$\boxed{3^{rd} \text{ six weeks}}$

(82) $\cos 75° = \cos(45° + 30°)$

$= \cos 45° \cos 30° - \sin 45° \sin 30°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

(83) $\tan \frac{\pi}{12} = \tan 15°$

$= \tan(60° - 45°)$

$= \frac{\tan 60° - \tan 45°}{1 + \tan 60° \tan 45°}$

$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$

$= \frac{-1 + \sqrt{3}}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$

$= \frac{-1 + \sqrt{3} + \sqrt{3} - 3}{1 - 3}$

$= \frac{-4 + 2\sqrt{3}}{-2}$

$= \boxed{+2 - \sqrt{3}}$

(84) $\sin \frac{11\pi}{12} = \sin 165^\circ$

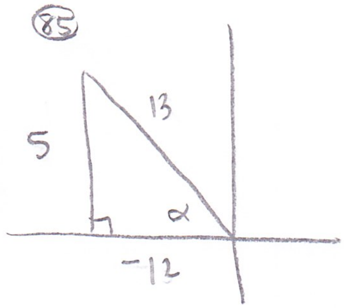
$= \sin(120^\circ + 45^\circ)$

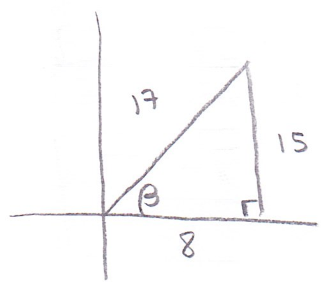
$= \sin 120^\circ \cos 45^\circ + \cos 120^\circ \sin 45^\circ$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + -\frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

(85)





(a) $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

$= \frac{5}{13} \cdot \frac{8}{17} + \frac{-12}{13} \cdot \frac{15}{17}$

$= \boxed{\frac{-140}{221}}$

(b) $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

$= \frac{-12}{13} \cdot \frac{8}{17} + \frac{5}{13} \cdot \frac{15}{17}$

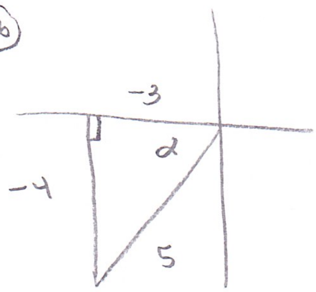
$= \boxed{\frac{-21}{221}}$

ⓒ $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$

$= \dfrac{\left(\frac{-5}{12} + \frac{15}{8}\right)}{\left(1 + \frac{-5}{12}\cdot\frac{15}{8}\right)}$

$= \boxed{\dfrac{20}{3}}$

(86)



ⓐ $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2\cdot\frac{-4}{5}\cdot\frac{-3}{5}$

$= \boxed{\dfrac{24}{25}}$

ⓑ $\cos 2\alpha = 2\cos^2\alpha - 1$

$= 2\left(\frac{-3}{5}\right)^2 - 1$

$= \boxed{\dfrac{-7}{25}}$

ⓒ $\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

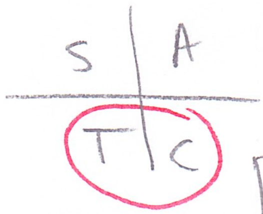
$= \dfrac{2\left(\frac{+4}{3}\right)}{1-\left(\frac{+4}{3}\right)^2}$

$= \boxed{\dfrac{-24}{7}}$

(87) $8 + \csc x = 2 \qquad 0^\circ \le x < 360^\circ$

$\csc x = -6$

$\sin x = -\frac{1}{6}$



$x = \sin^{-1}\left(-\frac{1}{6}\right)$

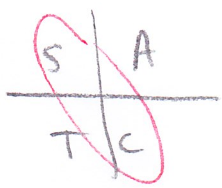
$\sin^{-1}\left(-\frac{1}{6}\right) + 360$

$\boxed{x \approx 350^\circ,\ 190^\circ}$

$\sin^{-1}\left(\frac{1}{6}\right) + 180$

(88) $\tan\theta + \sqrt{3} = 0 \qquad 0 \le x < 2\pi$

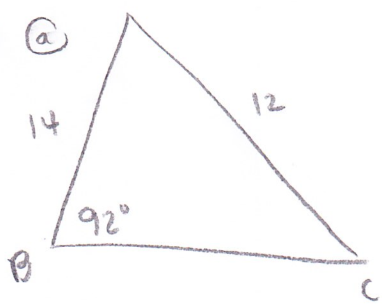
$\tan\theta = -\sqrt{3}$



ref $\angle\ \frac{\pi}{3}$

$\boxed{\frac{2\pi}{3},\ \frac{5\pi}{3}}$

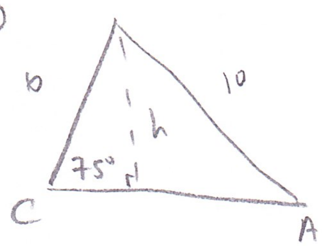
(89) (a)



$\boxed{0\ \Delta s}$

longest side isn't opposite the obtuse $\angle$

(b)

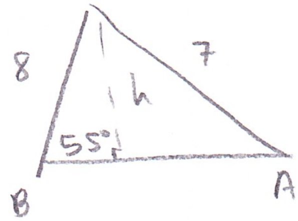


$h = 6 \sin 75^\circ$

$h \approx 5.8$

$\boxed{1\ \Delta}$

(c)

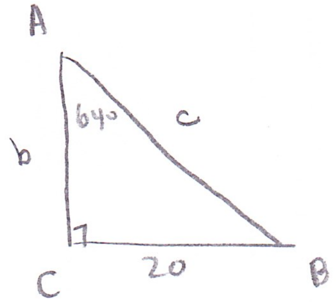


$h = 8 \sin 55°$

$h \approx 6.7$

$\boxed{2\ \Delta s}$

(90)



$\sin 64° = \frac{20}{c}$

$c = \frac{20}{\sin 64°}$

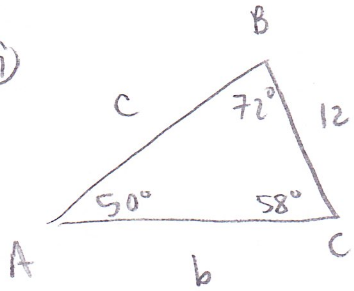
$\boxed{c \approx 22}$

$\tan 64° = \frac{20}{b}$

$b = \frac{20}{\tan 64°}$

$\boxed{b \approx 10}$

(91)



$C = 180 - 50 - 72$

$\boxed{C = 58°}$

$\frac{\sin 72°}{b} = \frac{\sin 50°}{12}$

$b = \frac{12 \sin 72°}{\sin 50°}$

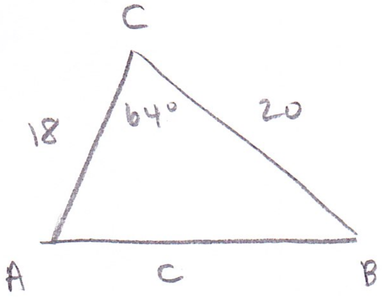
$\boxed{b \approx 15}$

$\frac{\sin 58°}{c} = \frac{\sin 50°}{12}$

$c = \frac{12 \sin 58°}{\sin 50°}$

$\boxed{c \approx 13}$

(92)



$$\sqrt{c^2} = \sqrt{18^2 + 20^2 - 2 \cdot 18 \cdot 20 \cdot \cos 64°}$$

$$\boxed{c \approx 20}$$

$$\frac{\sin B}{18} = \frac{\sin 64°}{c}$$

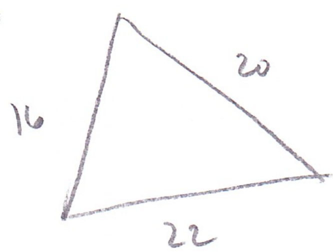
$$B = \sin^{-1}\left(\frac{18 \sin 64°}{c}\right)$$

$$\boxed{B \approx 53°}$$

$$A = 180 - 64° - B$$

$$\boxed{A \approx 63°}$$

(93)



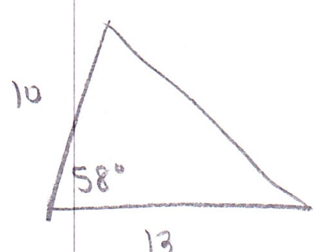
$$s = \frac{16 + 20 + 22}{2}$$

$$s = 29$$

$$K = \sqrt{29 \cdot 13 \cdot 9 \cdot 7}$$

$$\boxed{K \approx 154 \ u^2}$$

(94)



$$K = \frac{1}{2} \cdot 10 \cdot 13 \sin 58°$$

$$\boxed{K \approx 55 \ u^2}$$