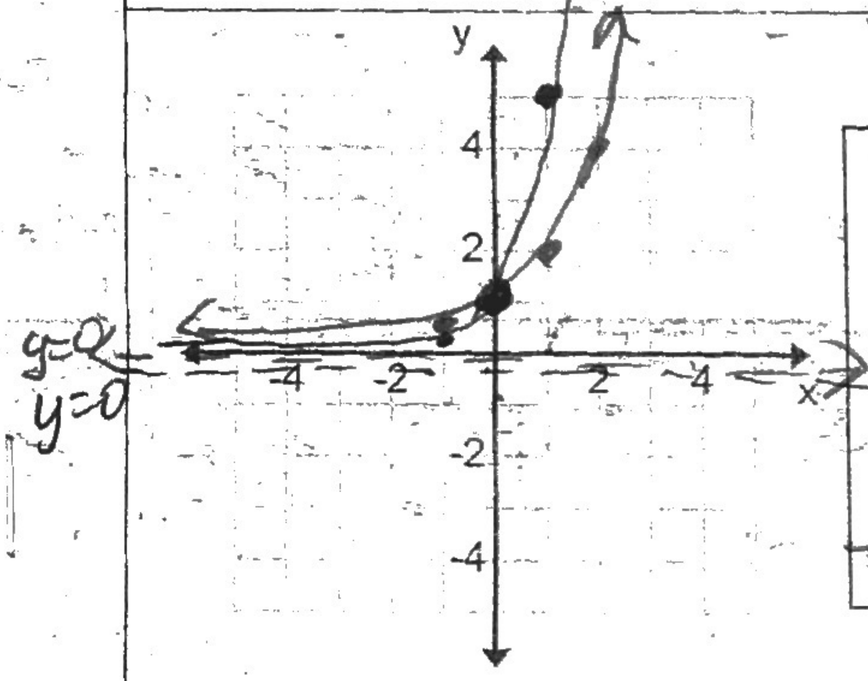
EXPONENTIAL GROWTH & DECAY

$$f(x) = A(B)^x$$ ← start, ← exponent, ← Rate

GROWTH Base > 1



y=0

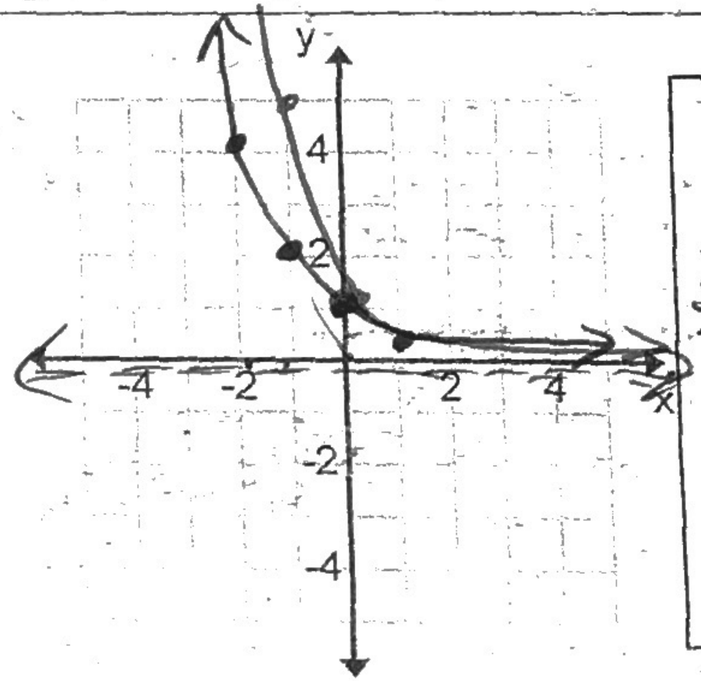
1) $f(x) = 2^x$

x	y
0	1
-1	$\frac{1}{2}$
1	2

2) $f(x) = 5^x$

x	y
0	1
-1	$\frac{1}{5}$

DECAY 0 < Base < 1



$f(x) = 2^{-x}$

1) $f(x) = \left(\frac{1}{2}\right)^x$

x	y
0	1
-1	2

$\left(\frac{1}{2}\right)^{-1} = 2$

2) $f(x) = \left(\frac{1}{5}\right)^x$

$y = 5^{-x}$

A) What happens to the graph as the base increases?	B) What is the y-intercept for each function? WHY?	A) What happens to the graph as the base increases?	B) What is the y-intercept for each function? WHY?
y increases faster.	1, anything to zero power = 1	decreases slower.	1 anything to zero power = 1
C) What are the x-intercepts? WHY?	**D) What is the domain of each function? What is the range?**	**C) What are the x-intercepts? WHY?**	**D) What is the domain of each function? What is the range?**
none HA $y=0$	D: $\mathbb{R}$ R: $(0,\infty)$ or $y>0$	none HA $y=0$	D: $\mathbb{R}$ R: $(0,\infty)$ or $y>0$

Transformation of Exponential Functions

$f(x) = V(B)^{(x-R)} + U$

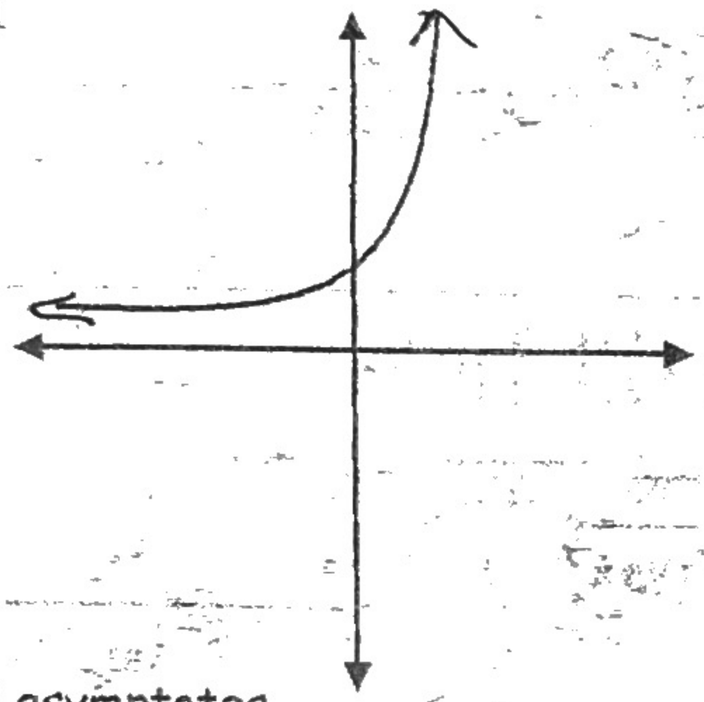
Describe each transformation from its parent function $f(x) = 2^x$

1. $y = 2^{(x-2)}$ Right 2	2. $y = 2^{x+4}$ left 4	3. $y = 2^x - 5$ Down 5	4. $y = 2^x + 1$ up 1
5. $y = 3(2)^x$ Vertical Stretch of 3	6. $y = -2^x$ Reflect across x-axis	7. $y = 2^{-x}$ Reflect across y-axis	8. $y = \left(\frac{1}{2}\right)^x$ $y = 2^{-x}$ Reflected across y-axis
9. $y = (-2)^x$ No Solution	10. $y = 3\left(2^{x+2}\right) - 1$ V. Stretch 3 Left 2 Down 1	11. $y = -\left(2^{x-6}\right) + 3$ Reflect across x-axis Rt 6 up 3	12. $y = 2^{-x} + 1$ reflect across y-axis up 1

GENERALIZATIONS FOR EXPONENTIAL FUNCTIONS

Growth

Graph of $y = a^x$, $a > 1$



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Intercept: $(0,1)$

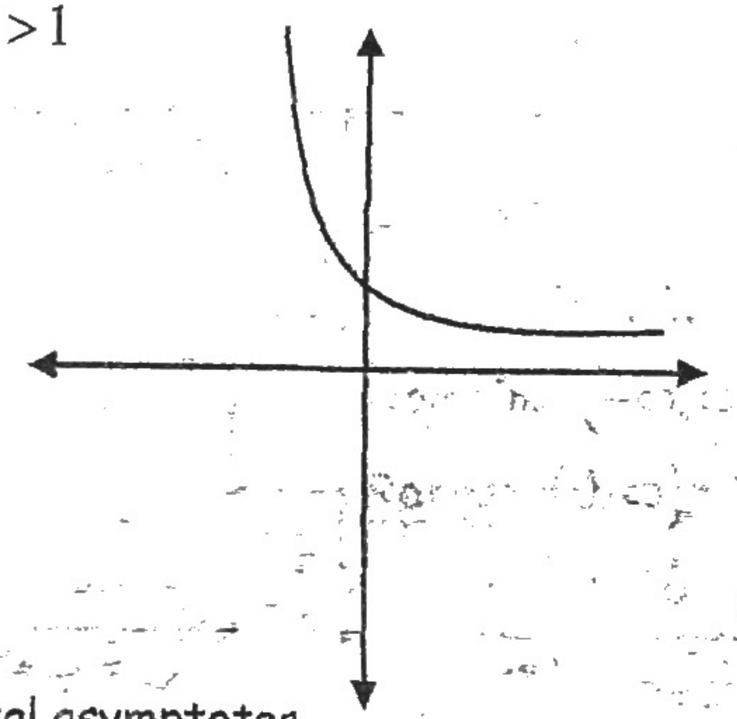
Increasing

x - axis is a horizontal asymptotes

$(a^x \to 0 \text{ as } x \to \infty)$

Continuous

Graph of $y = a^{-x}$, $a > 1$



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Intercept: $(0,1)$

Decreasing

x - axis is a horizontal asymptotes

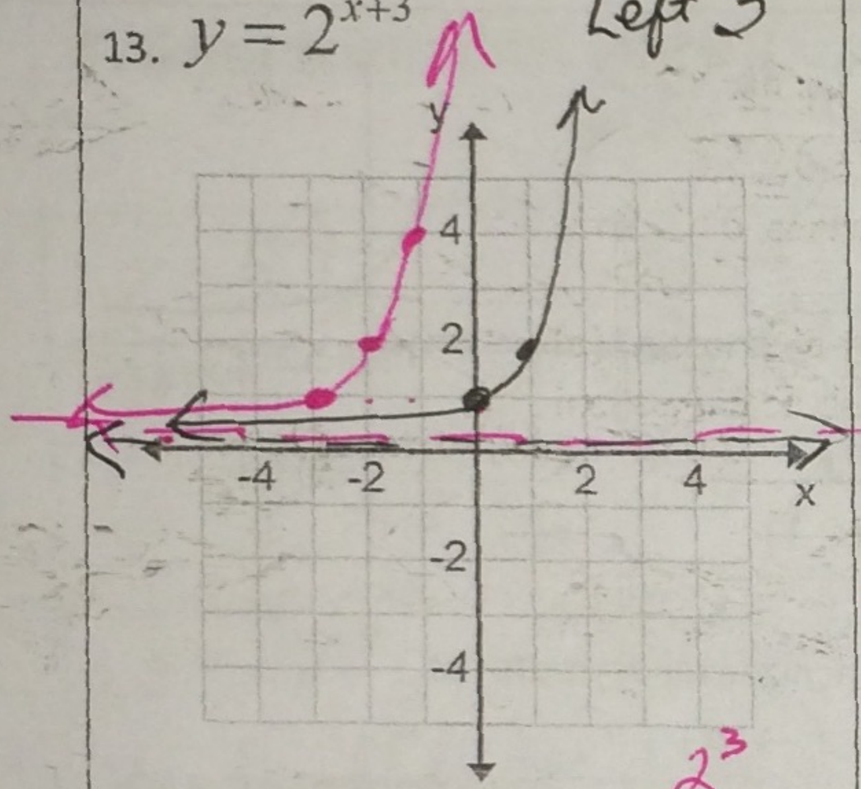
$(a^{-x} \to 0 \text{ as } x \to \infty)$

Continuous

Horizontal Shift $y = a^x$ to $y = a^{x-c}$	Vertical Shift $y = a^x$ to $y = a^x + c$
Reflection in x-axis $y = a^x$ to $y = -a^x$	Reflection in y-axis $y = a^x$ to $y = a^{-x}$

Transformations of Exponential Functions

13. $y = 2^{x+3}$ Left 3



2^3

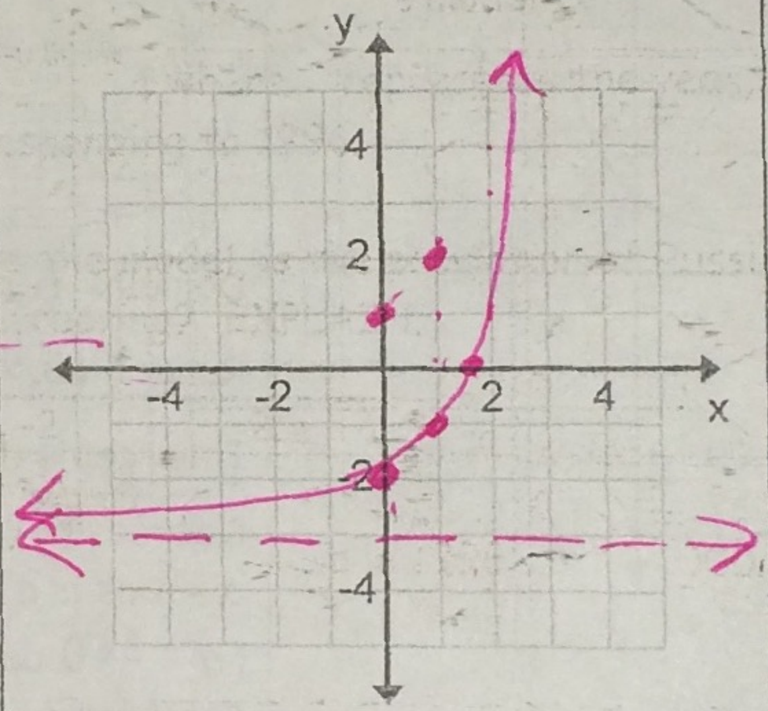
x-int: none y-int: 8

asymptotes: $y=0$

Domain: $\mathbb{R}$ $(-\infty,\infty)$

Range: $(0,\infty)$

14. $y = 2^x - 3$ Down 3



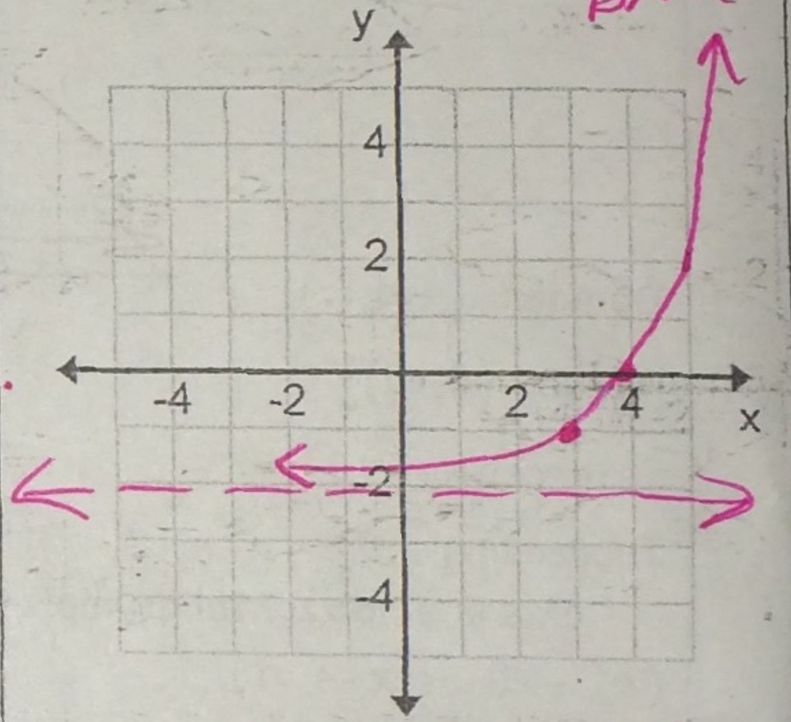
x-int: $x \approx 1.58$ y-int: -2

asymptotes: $y=-3$

Domain: $\mathbb{R}$

Range: $(-3,\infty)$

15. $g(x) = 2^{x-3} - 2$ Rt 3 Down 2



x-int: 4 y-int: $-\frac{7}{8}$

asymptotes: $y=-2$

Domain: $\mathbb{R}$

Range: $(-2,\infty)$

$0 = 2^x - 3 \quad y \geq -3$

$3 = 2^x \quad \log_2 3 = x$

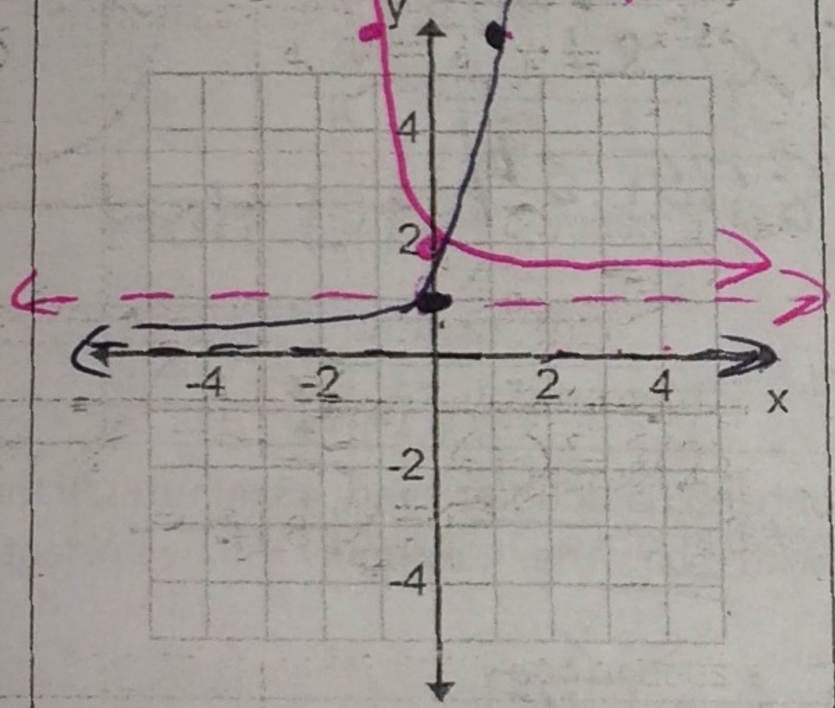
$y = 2^{-3} - 2$

$= \frac{1}{8} - 2$

Transformations of Exponential Functions

16. $y = 6^{-x} + 1$

Reflect across y-axis
up 1



x-int: none y-int: 2

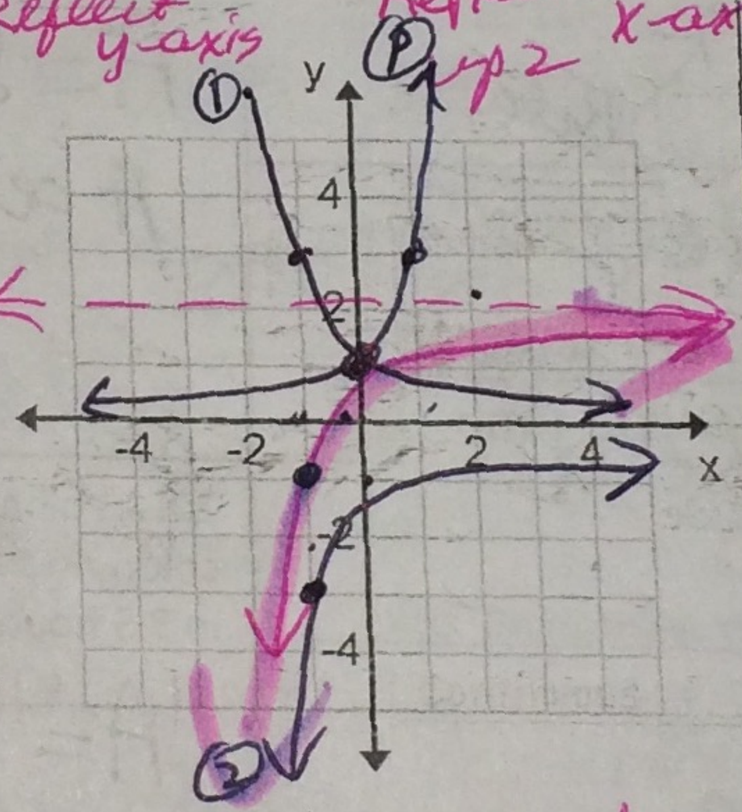
asymptotes: $y = 1$

Domain: $\mathbb{R}$

Range: $y > 1$

17. $f(x) = -3^{x} + 2$

Reflect y-axis
Reflect across x-axis
up 2



x-int: — y-int: 1

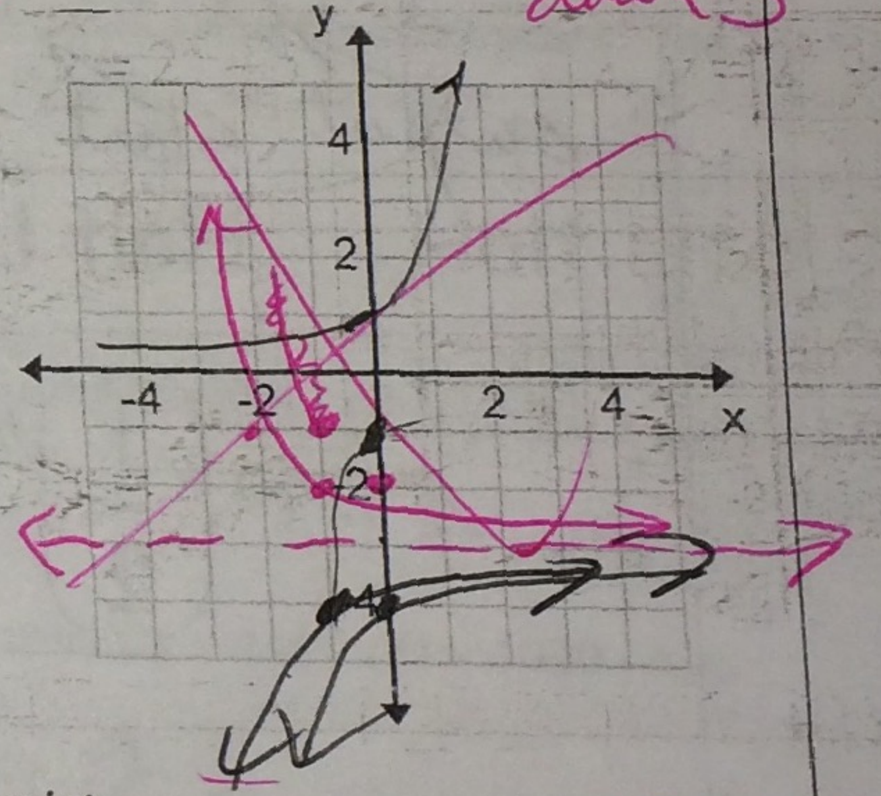
asymptotes: $y = 2$

Domain: $\mathbb{R}$

Range: $(-\infty, 2)$

18. $g(x) = -2^{x+1} - 3$

Reflect x-axis
left 1
down 3



x-int: _____ y-int: —

asymptotes: $y = -3$

Domain: $\mathbb{R}$

Range: $(-3, \infty)$

Compounding Continuously

$$A = Pe^{rt}$$

r ← decimal
t ← years

output: A
Start: P
Rate: r

Half Life

$$A = P\left(\frac{1}{2}\right)^{\frac{\text{actual time}}{\text{half-life time}}}$$

Doubling

$$A = P(2)^{\frac{\text{actual time}}{\text{doubling time}}}$$

19. A deposit of $500 is made in a trust fund that pays 2.8% interest compounded continuously.

What is the value of the account in 20 years?

$A = 500e^{.028(20)}$

$A \approx \$75.3\overline{3}$

20. A colony of bacteria doubles every 45 minutes. If th colony starts with 1 bacteria, how many will be present in 3.5 hours?

$A = 1(2)^{\frac{x}{45}}$

$A = (2)^{\frac{210}{45}}$

$A \approx 25$

Rate = (1 ± % decimal)

Applications of Exponential Functions

21. The population P (in millions) of Russia from 1996 to 2004 can be approximated by the model $P = 152.26e^{-0.0039t}$, where t represents the year, with $t = 6$ corresponding to 1996.

(a) According to the model, is the population of Russia increasing or decreasing? EXPLAIN.

Increasing Decreasing

(b) Find the population of Russia in 1998 and in 2000.

1998 : 147.52 million

2000 146.38 million

(c) Use the model to predict the population of Russia in 2010.

2010 140.78

Pop. in 1990 152.2

22. Let Q represent a mass of a radioactive radium (^{226}Ra) (in grams), whose half-life is 1599 years. The quantity of radium present after t years is

$$Q = 25\left(\frac{1}{2}\right)^{t/1599}$$

(a) Determine the initial quantity (when $t = 0$).

25 g

(b) Determine the quantity present after 1000 years.

16.2 g.

(c) Use a graphing utility to graph the function over the interval $t = 0$ to $t = 5000$.