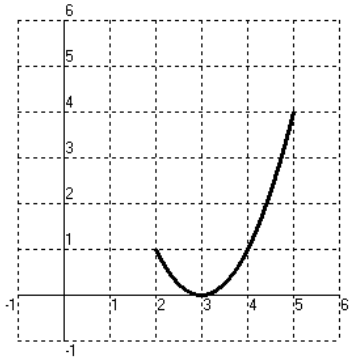
Pre-Calculus Notes Name: ____________________

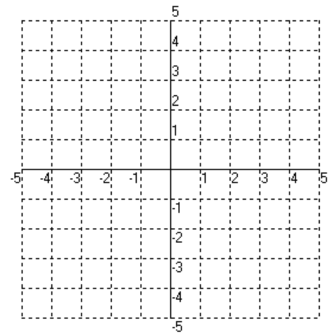
The Inverse Sine and Cosine Functions

DO WE REMEMBER HOW TO GRAPH A FUNCTION'S INVERSE?

1. Graph the inverse of the function below.



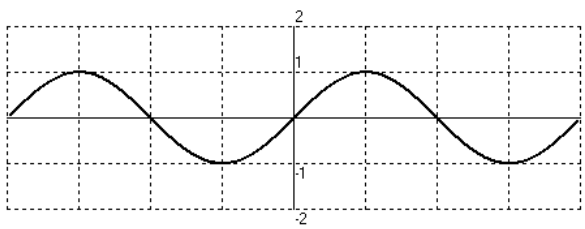
2. Graph the inverse of the line $y = \frac{3}{2}x - 4$.



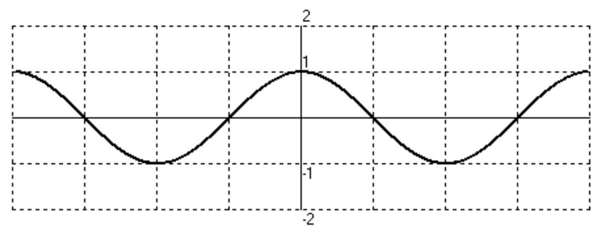
Why does this work?

So, let's graph the inverse of $y = \sin x$ and $y = \cos x$.

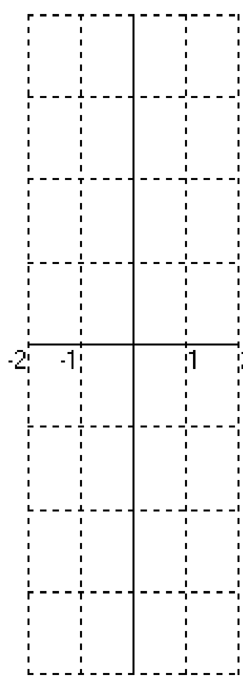
$y = \sin x$



$y = \cos x$

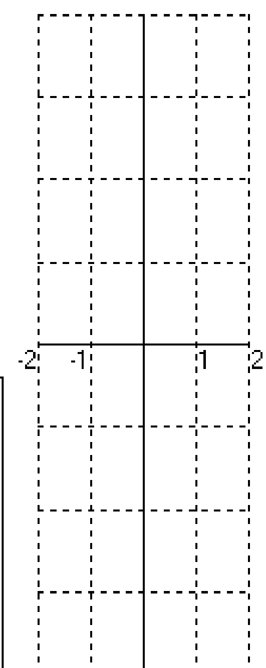


$y = \sin^{-1} x$



How can we restrict the domain of $y = \sin x$ so $y = \sin^{-1} x$ is a function?

$y = \cos^{-1} x$



How can we restrict the domain of $y = \cos x$ so $y = \cos^{-1} x$ is a function?

Now, let's make sure we **REALLY** understand inverse sine and cosine.

Since $\sin 30° = \frac{1}{2}$ ⟶ $\sin^{-1}$ __________ = __________

The ordered pair is (__________ , __________) ⟶ (__________ , __________)

So… If I say $\tan(a) = b$, then a is ____

and b is ____

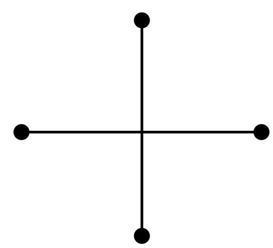
⟶ If I say $\tan^{-1}(c) = d$, then c is ____

and d is ____

You **STILL** need to remember your unit circle values.

If I ask you to find a trig value for ANY angle that terminates on an axis. ⟶ (Multiple of π or $\frac{\pi}{2}$)

What do you use?

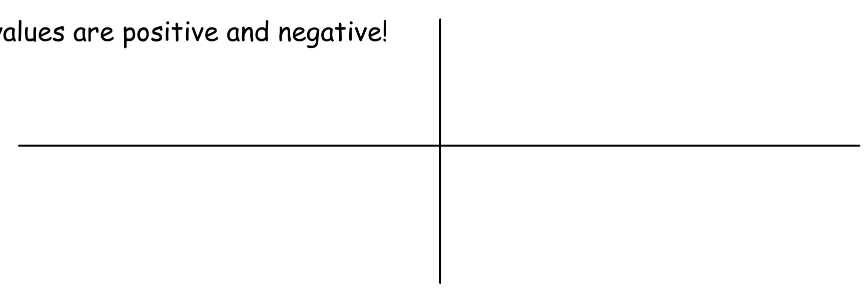


If I ask you to find a trig value for ANY angle that terminates in a quadrant. ⟶ (Multiple of $\frac{\pi}{6}, \frac{\pi}{4}$, or $\frac{\pi}{3}$)

What do you use? Do you remember HAND JIVE?

cos			
sin			
tan			

You also need to remember where the trig values are positive and negative!



Example 1: RECALL. Find the value for each of the following.

a. $\sin 60^\circ$	b. $\cos 300^\circ$	c. $\sin\left(\frac{5\pi}{4}\right)$
d. $\cos\left(\frac{3\pi}{2}\right)$	e. $\cos 45^\circ$	f. $\cos\left(\frac{5\pi}{6}\right)$

See how we input the angle and the output was a ratio? Well, for the inverse functions of sine and cosine, we input the ratio and the output is an angle. But not just any angle… an angle measure that falls in the range of the inverse function.

Example 2: Use the definition of the inverse to determine the EXACT value of each of the following.

a. $Sin^{-1}0$	b. $Sin^{-1}\left(-\frac{1}{2}\right)$	c. $Arcsin\ 1$
d. $Arccos\ \frac{1}{2}$	e. $Cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$	f. $Arcsin\ 1.5$

Example 3: Use the calculator to evaluate to the nearest tenth of a degree.

a. $Sin^{-1}0.258$	b. $Arccos\ 0.7644$	c. $Cos^{-1}(-0.56)$

Example 3: Evaluate to four decimal places.

a. $Cos^{-1}0.64$	b. $Arcsin\ (-0.91)$	c. $Sin^{-1}1.3451$

MEMORIZE… OR NOT.

These are shortcuts that cannot be used all of the time. So you would need to know when you can use them and when you cannot.

- $\sin(Arcsin\ x) = x$ AND $\cos(Arccos\ x) = x$ for all x where $-1 \le x \le 1$.
- $Arcsin(\sin x) = x$ for all x where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.
- $Arccos(\cos x) = x$ for all x where $0 \le x \le \pi$.

Example 4: Determine the EXACT value of each expression WITHOUT a calculator.

a. $\sin\left(Arcsin\ \frac{\sqrt{2}}{2}\right)$	b. $\cos\left(Cos^{-1}0.5\right)$
c. $Arccos\left(\cos\frac{7\pi}{6}\right)$	d. $\cos\left(Arcsin\left(-\frac{\sqrt{2}}{2}\right)\right)$
e. $Sin^{-1}\left(\cos\frac{5\pi}{3}\right)$	f. $\cos\left(Arcsin\left(\frac{\sqrt{2}}{2}\right)\right)$
g. $Arc\sin\left(\sin\frac{7\pi}{6}\right)$	h. $\sin\left(Cos^{-1}\left(-\frac{1}{2}\right)\right)$