

SOLVING SPECIAL EQUATIONS

Rational

3. $\left[\frac{10}{3} = \frac{4}{x} + 2 \right]$

$$10x = 12 + 6x$$

$$4x = 12$$

$$x = 3 \quad \checkmark$$

(*)

Radical

5. $(\sqrt{4x-3})^2 = (5)^2$

$$4x - 3 = 25$$

$$4x = 28$$

$$x = 7$$

$$\sqrt{4(7)-3} = 5$$

$$5 = 5$$

"u" substitution

Trinomials

9. $(x-7)^2 - 13(x-7) + 36 = 0$

$$u = x - 7$$

$$u^2 - 13u + 36 = 0$$

$$(u-4)(u-9) = 0$$

$$u = 4$$

$$u = 9$$

$$x - 7 = 4$$

$$x - 7 = 9$$

$$x = 11$$

$$x = 16$$

1. Mult. ALL TERMS by LCM: Get rid of the denominator

2. Solve Alg.

Check solutions

(*)

1. Isolate the Radical if possible

2. Square Both sides

3. Solve Alg

*Check solutions.

1. Replace "middle term" w/ "u"

2. Solve w/ Alg

3. replace "u" w/ middle term

$$2(x-7) \left[\frac{2x-9}{(x-7)} + \frac{x}{2} = \frac{5}{(x-7)} \right]$$

$$2(2x-9) + x(x-7) = 5(2)$$

$$4x - 18 + x^2 - 7x = 10$$

$$x^2 - 3x - 18 = 10$$

$$x^2 - 3x - 28 = 0$$

$$(x-7)(x+4) = 0$$

$$x = \cancel{7} \text{ OR } -4$$

$$\boxed{x = -4}$$

$$3. \frac{x-2}{5} \cdot \frac{3}{x+3} = \frac{24}{x^2+x-6}$$

$$5(x+3) + 3(x-2) = 24$$

$$5x + 15 - 3x + 6 = 24$$

$$2x + 21 = 24$$

$$2x = 3$$

$$\boxed{x = \frac{3}{2}}$$

$$4. \frac{5}{2x-1} = \frac{15}{x^2-1}$$

$$5(x^2-1) = 15(2x-1)$$

$$5x^2 - 5 = 30x - 15$$

$$5x^2 - 30x + 10 = 0$$

$$5(x^2 - 6x + 2) = 0$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{28}}{2}$$

$$= \frac{6 \pm 2\sqrt{7}}{2}$$

~~Extraneous Solutions~~

$$\boxed{x = 3 \pm \sqrt{7}}$$

$$\cancel{5/2}$$

6. $3\sqrt{x} - 5 = 13$

$\frac{3\sqrt{x}}{3} = \frac{18}{3}$

$(\sqrt{x}) = (6)^2$

$x = 36$

$(3\sqrt{x})^2 = (18)^2$

$9x = 324$

$x = 36$

7. $(\sqrt{2x+3}) = (x)^2$

$2x+3 = x^2$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = 3$ or ~~$x = -1$~~

$\sqrt{2(3)+3} = 3$

$\sqrt{9} = 3$

$3 = 3 \checkmark$

$\sqrt{2(-1)+3} = -1$

$\sqrt{1} = -1$

$1 \neq -1$

8. ~~$3\sqrt{x} - 5 = 13$~~

$(\sqrt{3x+7}) = (x+3)$

$3x+7 = x^2 + 6x + 9$

$0 = x^2 + 3x + 2$

$0 = (x+1)(x+2)$

$x = -1$ or -2

$\sqrt{3(-1)+7} = -1+3$

$\sqrt{4} = 2$

$2 = 2 \checkmark$

$\sqrt{3(-2)+7} = -2+3$

$\sqrt{1} = 1$

$1 = 1$

P7

$$10. \quad 3\left(\frac{w}{6}\right)^2 - 8\left(\frac{w}{6}\right) + 4 = 0$$

$$u = \frac{w}{6}$$

$$3u^2 - 8u + 4 = 0$$

$$(3u - 2)(u - 2) = 0$$

$$u = \frac{2}{3} \quad u = 2$$

$$\frac{w}{6} = \frac{2}{3}$$

$$\frac{w}{6} = 2$$

$$3w = 12$$

$$w = 4$$

$$w = 12$$

$$11. \quad 2x^4 + 7x^2 - 4 = 0$$

$$u = x^2$$

$$2u^2 + 7u - 4 = 0$$

$$(2u - 1)(u + 4) = 0$$

$$u = \frac{1}{2} \quad u = -4$$

$$x^2 = \frac{1}{2} \quad x^2 = -4$$

$$x = \pm \frac{1}{\sqrt{2}}$$

$$x = \pm \frac{\sqrt{2}}{2} \quad x = \pm 2i$$

$$12. \quad x + 7\sqrt{x} - 8 = 0$$

$$u = \sqrt{x}$$

$$u^2 + 7u - 8 = 0$$

$$(u + 8)(u - 1) = 0$$

$$u = -8 \quad u = 1$$

$$\sqrt{x} = -8$$

$$x = \cancel{64}$$

$$\sqrt{x} = 1$$

$$x = 1$$

$$64 + 56 - 8 = 0$$