Pre-Calculus Notes Name: Key

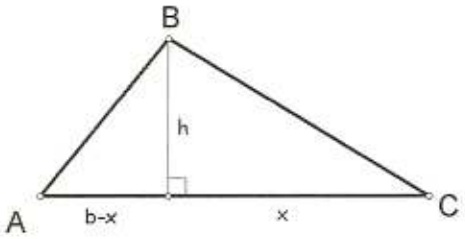
Section 6.2 – Law of Cosines DAY ONE

Remember the Law of Sines?

We used it to *find a side* given AAS or ASA.

We used it to *find an angle* given SSA.
(Note: we had to always check for the number of triangle on this one!)

Could we ***find a side*** given **SAS?**



Derivation of the Law of Cosines:

$c^2 = h^2 + (b - x)^2$

algebra magic

$c^2 = a^2 + b^2 - 2ab\cos C$

MEMORIZE: THE LAW OF COSINES

For ANY triangle ABC, where a, b, and c are the lengths of the sides OPPOSITE the angles with measures A, B, and C (respectively)...

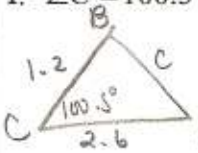
- $a^2 = b^2 + c^2 - 2bc\cos A$
- $b^2 = a^2 + c^2 - 2ac\cos B$
- $c^2 = a^2 + b^2 - 2ab\cos C$

Generally, since not every triangle is labeled A,B,C:
The length of a missing side = sum of squares of given sides - twice the product of the given sides and the cosine of included angle

(Note: we must have **SAS** to use this formula.)

Examples: Solve triangle ABC.

1. $\angle C = 100.5°$, $a = 1.2$, and $b = 2.6$



$c^2 = 1.2^2 + 2.6^2 - 2(1.2)(2.6)\cos 100.5°$

$c \approx 3.1$ STORE!

$\frac{\sin A}{1.2} = \frac{\sin 100.5}{c}$

$A = \sin^{-1}\left(\frac{1.2\sin 100.5°}{c}\right)$

$A \approx 23°$

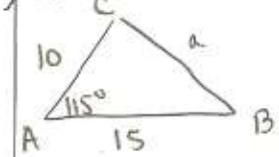
STORE!

$B = 180° - 100.5° - A$

$B \approx 57°$

2. $\angle A = 115°$, $b = 10cm$, and $c = 15cm$

$a^2 = 10^2 + 15^2 - 2(10)(15)\cos 115°$



$a \approx 21$ STORE!

$\frac{\sin 115°}{a} = \frac{\sin B}{10}$

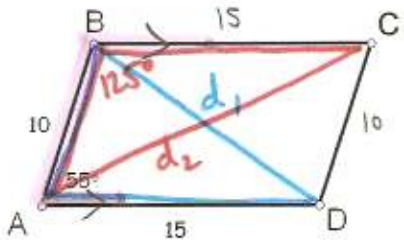
$B = \sin^{-1}\left(\frac{10\sin 115°}{a}\right)$

$B \approx 25°$ STORE!

$C = 180° - 115° - B$

$C \approx 40°$

Example 3: Given the following parallelogram, find the measures of the other angles and the two diagonals.



$B = 125°$, $D = 125°$, $C = 55°$

$\sqrt{(d_1)^2} = \sqrt{10^2 + 15^2 - 2(10)(15)\cos 55°}$

$d_1 \approx 12.4$

$\sqrt{(d_2)^2} = \sqrt{10^2 + 15^2 - 2(10)(15)\cos 125°}$

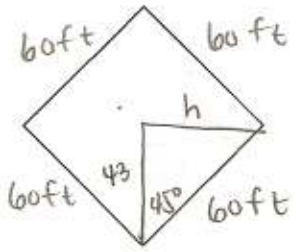
$d_2 \approx 22.3$

Example 4: WORD PROBLEM.

The pitcher's mound on a women's softball field is 43 feet from home plate and the distance between the bases is 60 feet. (The pitcher's mound is not halfway between home plate and second base.) How far is the pitcher's mound from first base?

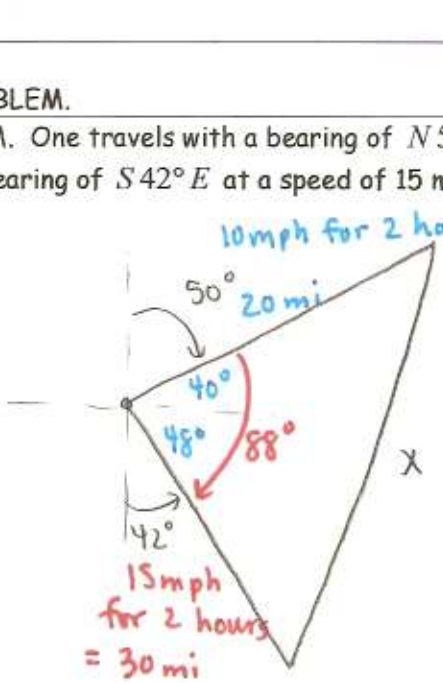
$\sqrt{h^2} = \sqrt{43^2 + 60^2 - 2(43)(60)\cos 45°}$

$h \approx 42.43$ feet



Example 5: WORD PROBLEM.

Two ships leave port at 1P.M. One travels with a bearing of $N\,50°\,E$ at a speed of 10 miles per hour. The Other ship travels with a bearing of $S\,42°\,E$ at a speed of 15 miles per hour. At 3 P.M., how far apart will the ships be?



$X^2 = 30^2 + 20^2 - 2(30)(20)\cos 88°$

≈ 35.5 miles